CONSISTENT MODELLING OF INDEX AND VOLATILITY DERIVATIVES WITH A RANDOM FIELD LOCAL VOLATILITY MODEL

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Abstract

We propose a unified framework for the joint modelling of an index and its local volatility surface. This model enables index and volatility deriva- tives to be priced consistently and reproduces empirical features of the eq- uity market, including the strong negative correlation between the changes of SPX and VIX, any observed term structure of variance swap rates, and the volatility skews and term structures observed in SPX and VIX op- tions. In the framework of the Heath-Jarrow-Merton (HJM) philosophy, we start from an initial collection of index options and their associated local volatility surface, and show how to construct the arbitrage-free evo- lution of this local volatility surface from a Gaussian random field. This arbitrage-free condition is similar to that of the dynamics of the forward interest rate in the original HJM model. We also present the relation- ship between the local volatility model and the variance rate model. In the end, we show how to implement our model numerically by the Monte Carlo simulation and the finite difference method, and how to simulta- neously calibrate to the prices of SPX options and VIX options across strikes and maturities.

Keywords: volatility derivatives; variance term structure; VIX; stochastic local volatility; Gaussian random field; HJM model.