AUTOMATED VAR MODEL WITH ADAPTED WEIGHTING IN ABSOLUTE RETURN

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Abstract: Absolute Return (AR) refers to an investment strategy which does not depend on a particular Benchmark. This strategy aims at making positive return regardless of market volatility. So AR does not provide open stock positions because it is based on the possibility to withdraw invested money at any time. In this domain, there does not exist literature about measurement risks in a traditional way such as Value at Risk. The purpose of present study is to palliate this lack of information. To this aim, two approaches are developed to estimate the Value at Risk for short time samples: one with equi-probability losses and another one with multi-probability weighting losses. The two corresponding models are compared to estimate which one is the most reliable in term of risk (or most respectful of market reality). Finally, a stress test has been made to test the robustness of the two models.

Keywords: Absolute Return, VaR, Level of Confidence, Weighting, Risk Assessment, Probability Violation, Decay Factor (λ)

I. Introduction

Nowadays, the financial and economic world does not anymore inspire full confidence to investors. After 2008 Subprime Crisis, after the Greek crisis which shacked the entire European continent, and the Chinese Stock Market Crash in 2015, financial and stock markets do not represent a safe investment any more. Estimating loss of financial investments is now the crucial task in the market risk management inside current global economy. The importance of such task is even more critical in emerging financial markets where the conjunction of fluctuations in the volume of hot money from international portfolio investments and hedge funds, of unstable regulatory and political environment, and of lack of informational efficiency creates high volatility and extreme variations in the returns. In this particular context, different strategies have been proposed to reduce exposition to risk since early optimization methods [1,2] and evaluation of risk has been developed for best strategic choice [3-6]. Various “protecting” methods have been discussed [7,8] and the Value-at-Risk (VaR) is now the measure for testing and comparing various possible approaches [9-14] despite some limitations [15]. In particular, to protect investments against large fluctuations, Absolute Return (AR) strategies show interesting advantage [16-18]. More advanced analysis is developed with Extreme Value Theory (EVT) [19-20] and Filtered Extreme Value Theory (FEVT) [21-23]. AR strategies represent the return of an investment on an asset at some point without particular benchmark on the market. So AR investment would then appear as *the* solution against relative value approach, as it measures absolute return according to the market, whether bearish or bullish. As a solution to hedge market risk by considering an anti-benchmark strategy, AR represents in a way a safe manner to invest. However, without taking into account the market and its fluctuations, the AR investor does not integrate trends in his modus operandi. Following an AR strategy just forces him to invest on little samples of time, in order to avoid bad consequences related to possible market downturns unpredictable in his investor point of view since he has no benchmark. For this reason, to address the problem, AR investors use the technique of portfolio diversification consisting in favoring weakly correlated assets. Nevertheless, such technique does not provide coverage against intrinsic volatility related to each asset, so investment in AR remains weak, and spreads over a little sample of time.

The goal of present project is to shed some light on investments in AR for better determination of higher return from such investments. To proceed, the study will be based on two different methods using the VaR. First one is an application of traditional calculation of VaR applied to AR and is based on VaR expression [24]. There are two different methods to approach the VaR: 1) The parametric method, which requires to take into account the value P of the portfolio, the value $Z_{conf\%}$ defined by log normal law $P(Z > Z_{conf\%}) = Conf\%$, and daily volatility σ_j. Then one gets for one day the expression VaR(conf%,1day)

= P $Z_{conf\%}$ □, and for N days one gets VaR (conf%,N) = $N^{1/2}$ VaR (conf%,1); 2) The historical method which uses portfolio loss L defined as the difference between portfolio current market price p and its price at a given horizon P. It then gives L = p □P. This method, after chosing the confidence percentage, allows calculate a VaR for each sample interval □t,and to analyze favorable samples to AR investments. Here historical method is used because parametric one is not adapted to the non-linear instruments, that is to say those which do not have a linear payoff function in terms of market value like options, callable/puttable bonds and swaptions. This restricts portfolio analysis to only take account of linear instruments. Moreover, parametric method is not adapted to non-normal distributions or thick tails, whereas historical method can work with all the instruments (except only the derivatives), and does not take into account the distribution. Finally, the historical method is based on true facts, whereas parametric one is more speculative and anticipative, a possible reason why investors have more confidence in past history.

The second model still leans on historical values, in this case on Historical Simulations (HS), more precisely on future foresight by simulation, or the construction of a Cumulative Distribution Function (CDF) of return on investment through time. Associated to HS, an equivalent of Exponentially Weighted Moving Average (EWMA) is used, the only difference is in the use of a system of powers and not a system of exponentials. The goal of this level-headedness is to detect the slightest steps of the VaR compared with its mean. Their association leads to an allowance of a decreasing weight to each scenario, then to the construction of an empirical distribution. The goal here is to reduce the risk related to the portfolio, then to have a low probability of percentage violation. The weights λ^K are obtained, K being the number of simulations observed, and λ the decay factor related to the function slope.

These two models have been totally automated in a VBA program. Moreover, a stress study has been made corresponding to extreme market conditions to test algorithms robustness.

II. Risk Approximation by VaR Calculation

To calculate the risk, the method of VaR has been used. A VaR at q% indicates that for quantile value α = q

$$[L \leq v] = \alpha \tag{2}$$

with VaR(q) = v. The percentage q% is the level of confidence in the simulation and the term L is considered as a random variable, with each value of loss supposed to have the same probability equal to 1/N with N the number of loss values. So the purpose is to find the value of satisfying (2). With a general series of Δ mn intervals, S_t is the market value at time t and $S_t + S_\Delta$ is a random variable. To know the risk when investing at t + □it is necessary to have all values indexed on value t.

For the project, attention has been focused on different forex exchange, EUR/USD and EUR/JPY with Δ = 5mn duration. In each period Δ, there are an opening value, a closing value, the highest value and the lowest value. To estimate the VaR, the average of highest and lowest values is first calculated to obtain one unique value endorsing the period <X> = [x_+ + $x_□$]/2. Ranking all values by chronological order, the return represents the ratio of loss or gain in the last Δ mn interval:

$$R(.): \Omega \longrightarrow \{R_1, R_2, \ldots, R_n\} \;\; ; \;\; R_t = [x_t \square x \;\;]/x_t \;\; \square 1 \tag{3}$$

In turn this gives

$$S_{t+\Delta}(.): \Omega \longrightarrow \{S_1, S_2, \ldots, S_M\} \quad ; \quad S_m = x_T(1 + R_t) \tag{4}$$

x_T is most recent value fixed for all S_m, m □ [1,..M]. The ri profits and loss P&L given by:

$$P\& = S_m - x_T \tag{5}$$

Finally, the portfolio loss L(1) for one period is given by

$$(1) = \max(0; -P\&L) \tag{6}$$

To find VaR(q), the VaR at q%, L(.) is ranked from 0 to highest value and the value L(q) for q^{th} percentile is taken. The method has been automated to calculate many VaR and to increase confidence in the result. All VaR are obtained by fixing the most recent value so the calculations need to be started again, with

a new fixed value. They are performed with VBA program, using average R_t and return S_m which calculates P&L and L.
A stress test has also been done, and it is noticed that the violation probability is always the same. Here, the violation probability VP is on (1 □ q)% and is gi
larger than VaR. The variation of EUR/JPY currencies is shown on Figure **1** with calculated VaR for q = 95%. Except few times of sudden loss, the VaR curve is representative of gain and loss.

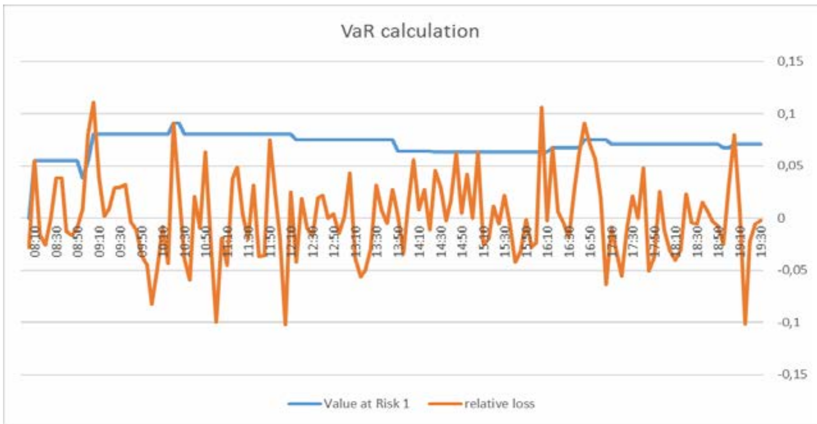

Figure **1**. Relative Loss and VaR vs Time for q = 95%

III. Improved Model with Weighted VaR

As indicated before historical values look more reliable for investors because the trend from this set of market values did already happened. Hence the importance of using historical value in a correct way. The purpose of this part is to improve the first method of VaR (q) calculation. In the first method all losses have been supposed equi-probable:

$$\text{Loss: } \Omega \longrightarrow \{\lambda1, \lambda2, \ldots, \lambda n\} \quad \text{and} \quad \forall k\varepsilon[|1, n|], \quad P(k) = 1/k \tag{7}$$

In this part, all past market values have different probabilities. According to Historical Simulation with Component Weight and Ghosted Scenario a couple of coefficients can be introduced to transcribe in another way historical values and the probability. Thus, probabilities are given by, see Figure **2**:

$$\text{Loss: } \Omega \longrightarrow \{\lambda1, \lambda2, \ldots, \lambda n\} \text{ and } \forall k\varepsilon[|1, n|], \quad P(k) = C\lambda^k \quad \text{with } 0 < \lambda \leq 1 \text{ and } C \in R \tag{8}$$

C and λ are the two coefficients which are automatically calculated with VBA. An important point to underline is that coefficients have to be less than 1. This condition implies that as k increases the probability becomes lower. These probabilities can be allocated to different historical market values and constitute a sort of weighting : higher k index corresponds to older historical values.

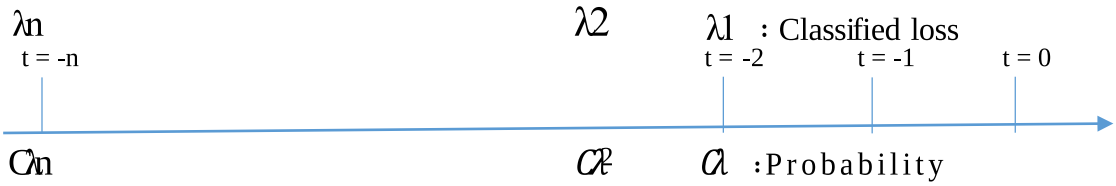

Figure **2**. Distribution of Losses and Probability vs Time

To adapt this theory to Absolute Return strategy, the calculation should find a correct λ to keep violation probability VP (the percentage of effective Losses superior to VaR) under the fixed value (1 □ q)%. This can be calculated at the end of the day (real value at 5.00pm to calculate value at 5.05pm with □ = 5mn). Best conditionsare:

$$VP \leq (1-q)\% \, ; \qquad R_l = \frac{effective\ Loss}{VaR} = 1 \tag{9}$$

R_l needs to be close to 1 because the VaR(q) represents, at time t, the maximum possible Loss at q%. In fact to have small enough value of VaR and not to freeze a too large amount of capital, R_l has to be large enough. In reality equality $R_l = 1$ is not satisfied because the real value of next effective Loss cannot be calculated by the program.
For calculating the coefficient C, all $C\lambda^k$ have to be added. Because the sum of probabilities is 1 one gets $(\lambda^1+\lambda^2+\cdots+\lambda^n)=1$ from which

$$C = \frac{1-\lambda}{\lambda(1-\lambda^n)} \qquad (10)$$

With the VaR program, different values of lambda can be tested to satisfy (9). When a correct □ is fixed C is calculated by (10) and VaR is obtained from VBA program, see Appendix.
To estimate VaR, the program calculate cumulative sum of probabilities:

$$q_1 = \ (1) \ ; \ q_2 = P(2) + q_1 \ \ldots \ q_k = P(k) + q_{k-1} \ ; \ q=1 \qquad (11)$$

For a fixed □th classified loss where k, in order to respect the most (9), the pr
is the qth percentile of q, see Figure **3**.

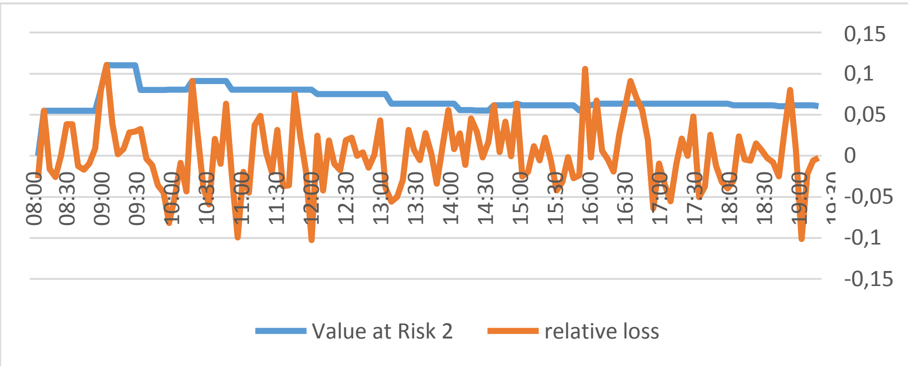

Figure **3**. Relative Loss and VaR from 2nd Model vs Time for q = 95%

It can be seen that improved model predicts an increase of relative loss at the beginning of the day (about 8.55 am). At the end of the day the VaR still stays high whereas the relative loss is low. But at about 19.10 pm a peak of loss occurs and justifies the high value of VaR.

IV. Comparison between VaR and Weighted VaR Models

When plotting first and second estimation of VaR vs. time, see Figure **4**, weighted VaR model gives more importance to the most recent data

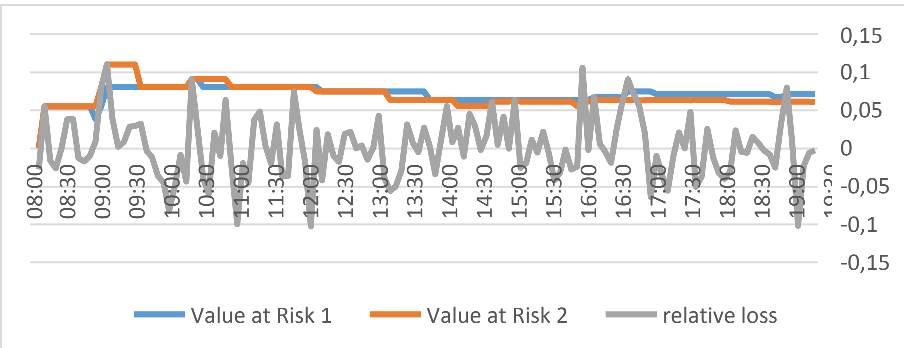

Figure **4**. Relative Loss, VaR and Weighted VaR vs. Time for q = 95%

The first peak of loss of the day has more impact on the weighted VaR than on VaR first model. Weighted VaR reaches the peak first and remains high whereas first VaR stays relatively low even if the loss has increased. Moreover, weighted VaR is closer to real loss as time goes on, as there are more stored data in memory. The results of the two discussed VaR models are summarized on Table **I**

	R_l	VP
VaR	.5843	5.7554
Weighted VaR	.6489	7.1924

Table **I**. Synopsis of Results for VaR and Weighted VaR Calculations for q = 95%

The ratio R_l measures VaR precision in average for the whole studied period. When it is close to 1, the VaR is not far from the effective loss. As the ratio with weighted VaR is higher than first one, weighted VaR calculation is more precise. Violation probability VP is the probability that the VaR failed in the prediction of expected loss. First probability is better with 5.7554% of violation, a good value not far from expected one (about 5%).
A stress test has also been done to test models robustness. In Forex rate curve of EUR/JPY exchange, a sudden fall has been simulated at 17.30 pm, see Figure **5**.

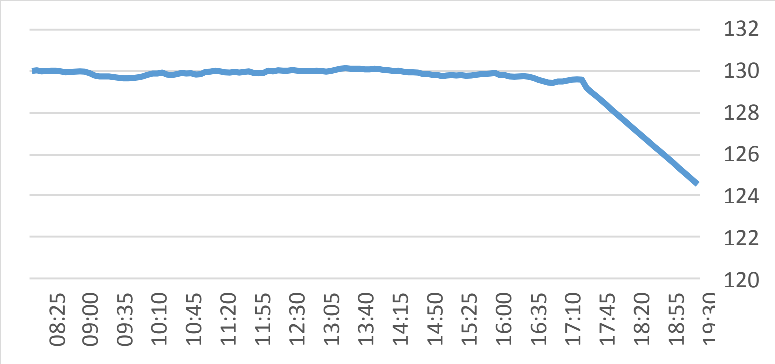

Figure **5**. EUR/JPY Exchange Rate vs Time with Stress Test at 17.30pm

The fall involves an increase of loss. Both models react to this increase at same level, see Figure **6**. The difference is that weighted VaR reacts faster than first VaR. The reason is that weighted model gives relatively more importance to the last minutes. The two models are good but second one reacts better to unexpected large variation.

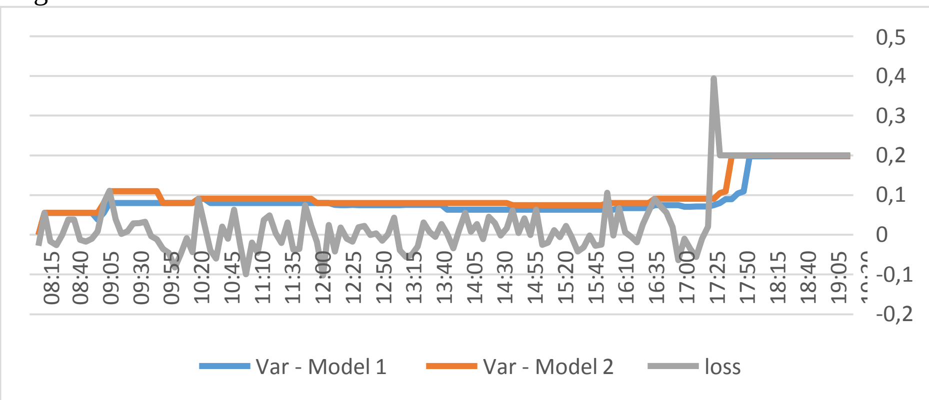

Figure **6**. Relative Loss and VaR from the 2 Models under Stress of Figure **5** vs Time

V. Conclusion

This paper shows that it is possible to adapt to AR the method of Value-at-Risk calculation specific to relative value processes. The VaR has been calculated with two different methods, each one dealing with relevant tools of AR. The first one is a classical calculus method taking account of the Loss of

historical value with same probability. The second method introduces a system of weighting of each scenario to note the impact of recent events on the fluctuation of the asset itself.
The risk inherent to short sampling of time, which is specific to investments in AR, has been calculated. The weighted model has a ratio closer to one, with the meaning that VaR value is closer to the reality. This could be expected because the second method depends on the most recent values. Using risk calculation, it is thus possible to find the best moment to invest in a day. More generally, knowing all VaR, a method can be set up to predict best time for investment. Furthermore, risk could be calculated with parametric models. Finally the two analyzed methods give equivalent performances but the weighted VaR is more accurate in absolute return.

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Appendix : VBA Program for Weighted VaR Model

```
For i = nbData to 1, step-1
    For j = I to nbData
        Sm(i,j) = value(i) * (1 + return (j))
        P&L(i,j) = Sm(i,j)-value(i)
            If P&L(i,j) <0 then
                Loss(i,j) = -P&L(i,j)
            Else
                Loss(i,j) = 0
            End if
    Next j
    For k = I to nbData-1
        For l=i to nbData-1
            If Loss(i,l)>Loss(i,l+1) then
                Tmp = Loss(i,l+1)
                Loss(i,l+1)=Loss(i,l)
                Loss(i,l)=tmp
            End if
        Next l
    Next k
Ka(i,i) = 1
Lam_ka(i,i) =λ^ka(i,i)
Sum(i)=lam_ka(i,i)
    For j=i+1 to nbData
        Ka(i,j) = ka(i,j-1)+1
        Lam_ka (i,j) = λ^ka(i,j)
        Sum(i)=sum(i)+ lam_ka(i,j)
    Next j
C(i)=1/sum(i)
Proba(i,i)=C(i) * lam_ka(i,i)
Sum_proba(i,i)=proba(i,i)
    For j=i+1 to nbData
        Proba(i,j)=C(i) *lam_ka(i,j)
        Sum_proba(i,j)= sum_proba(i,j-1) + proba(i,j)
    Next j
Indice(i) = i
    Do while (sum_proba(i, indice(i))<0.95)
        Indice(i) = indice (i)+1
    Loop
VaR(i) = Loss(I,indice(i))
    Next i
```