CALCULUS STUDENTS AND INFINITY: PERCEPTIONS AND MISCONCEPTIONS

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Abstract

Infinity has been a debatable subject throughout history. Infinity may be seen as a mathematical idea that causes various obstacles to students due to the duality of its meaning, as an object and as a process (Monaghan, 2001). Originally, the word apeiron (infinity), in ancient Greek, stood for unbounded, infinite, indefinite, or undefined. Aristotle separated the concept in two different aspects- potential and actual- that correspond to the ways of looking at infinity- as a process or as an object (Sacristàn & Noss, 2008; Tirosh, 1999). Pythagorus, on the other hand, had a different outlook of infinity. Zeno, then, formulated his paradoxes by mixing finite reasoning with infinite and limiting processes. In Euclid's The Elements, the very definition of a point, A point is that which has no part, invokes ideas of the infinite divisibility of space. This study aims to examine calculus students' understanding of infinity. Calculus is based on the idea of limits, and limits require their opposite — that which is not bounded. Newtonian calculus finesses the problem of an unfounded infinity by devising a method for recognizing and predicting infinity effects. Firstly, this study aims to examine the perceptions of calculus students regarding the concept of infinity. In particular, the two aspects of the concept- as a process or as an object - are examined through the definition and participants' responses. Secondly, misconceptions that participants have during the comparison of infinite sets or numbers with infinite decimals will be discussed. Finally, the impact of different representations in the comparison of infinite sets will be investigated. Our study involved 50 students in an Advanced Calculus class. Data was collected through a questionnaire. The present study identified misconceptions that students have related to the concept of infinity. This could help teachers later on devise ways to overcome these misconceptions. Recommendations include introducing applications of infinity in real life, and explaining infinity through its two aspects: the object and the process, and distinguishing each of the aspects by examples.

Keywords: Infinity; Student Perceptions; Student Misconceptions; Calculus