INVESTIGATING THE EMPIRICAL LINKAGE OF STOCK RETURNS, RETURN VOLATILITY AND TRADING VOLUME WITH AN ASYNCHRONOUS REGIME SWITCHING GO GARCH

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Abstract

This paper suggests an asynchronous Markov regime switching generalized orthogonal GARCH (*ARSGO*) model to study the relations among stock return, stock volatility, and trading volume. The contribution of this paper is twofold: first the proposed *ARSGO* is a *GO* GARCH such that different financial variables are governed by different state variables with the dependence of switching captured by a synchronization factor. Second, with the proposed *ARSGO*, the one-step estimation of stock returns, return volatility and trading volume under regime switching is feasible. In this paper, ARSGO is applied to investigate simultaneously the contemporaneous and causal relations between stock returns, return volatility and trading volume for Hang Seng and S&P 500 index.

Empirical results show that for both Hang Seng and S&P 500 index, unconditional mean returns are lower in the higher volatility state and mean percentage changes in trading volume are higher in the high volatility state. The one-period lagged index returns and percentage change in trading volume exhibit positive impacts on their current values in the high volatility state. Higher percentage change in volume last period causes higher current return for Hang Seng index but lower current return for S&P 500 index only in the high volatility state. The one-period lagged index return has a significant negative impact and a significant positive impact on current trading volume in the high and low volatility states, respectively for both indices. The volatility persistence tends to be more pronounced in higher return or trading volume volatility regime. It is also found that no significant impact of lagged percentage changes in trading volume on current return volatility. The average correlations between return and percentage changes in trading volume for Hang Seng index are all higher than that of S&P 500 index in each regime combinations and the extent of desynchronization between stock return and trading volume is higher in S&P 500 than in Hang Seng.

Keywords: Asynchronous Markov switching; *GO* GARCH; stock return; return volatility; trading volume

I. Introduction

A considerable body of literature has developed that empirically explores the contemporaneous relation between stock returns and trading volume (Epps, 1975; Richardson et al, 1986; Harris, 1987; Karpoff, 1987; Jain and Joh, 1988). Existing finance literature seems to suggest a positive return-volume relation but inconsistent conclusions still remain. As mentioned by Chuang et al. (2009), the dynamic (causal) relation between return and volume is more informative as far as prediction and risk management are concerned. Since the 1990s, the focus has moved to investigate dynamic relation and most of these studies investigate the causal relation with bivariate vector autoregressive (VAR) models and Granger causality tests (Hiemstra and Jones, 1994; Saatcioglu and Starks, 1998; Chordia and Swaminathan, 2000; Lee and Rui, 2002; Statman et al., 2006; Eleanor Xu et al., 2006; Griffin et al., 2007; Hutson et al., 2008; Chuang et al. 2009; Chen, 2012). The causality between stock return and trading volume, however, remains controversial. This is not surprising because the relation between past returns and trading activity depends on a number of factors that change through time and differ across countries (Griffin et al., 2007).

For instance, some articles suggest a unidirectional causality between trading volume and stock returns. Statman et al. (2006) investigating the monthly data in NYSE/AMEX shares from 1962 through 2001 and show that market-wide trading activity is positively correlated to past shocks in market return. Based on the sample of 46 developed and developing countries, Griffin et al. (2007) also document a strong positive relation between turnover and past returns especially in countries with high levels of corruption, with short-sale restrictions, and in which market volatility is high. In contrast, however, Saatcioglu and Starks (1998) examine the stock price-volume relation in a set of Latin American markets and fail to find strong evidence on stock price changes leading volume. Instead, they find that volume seems to lead stock price changes. Lee and Rui (2002) investigate the dynamic relations between stock market trading volume and returns of the three largest stock markets: New York, Tokyo, and London and find that trading volume does not Granger-cause stock market returns on each of three stock markets. Still others document a bidirectional returns-volume causality. Hiemstra and Jones (1994) apply both linear and nonlinear Granger causality tests to examine the dynamic relation between daily Dow Jones stock returns and percentage changes in New York Stock Exchange trading volume and find evidence of significant bidirectional nonlinear causality between returns and volume. Chuang et al., (2012) investigate the returns-volume relation for ten Asian stock markets and find that there is a positive bi-directional causality between stock returns and trading volume in Taiwan and China.

More sophisticate models have been applied to investigate the causal relation between stock returns and trading volume recently. McMillan (2007) apply a logistic smooth-transition model (*LSTR*) to investigate the dynamic of UK, US, France and Japan equity returns and conclude that using lagged volume as a threshold improves the performance of return forecasts. Chuang et al. (2009) investigate the causal relations between stock return and volume for three major indices, NYSE, S&P 500 and FTSE 100, based on quantile regressions and find that the causal effects of volume on return are usually heterogeneous across quantiles and those of return on volume are more stable. Chen (2012) investigates the relation of S&P 500 price index and trading volume with a two-state Markov regime switching vector autoregressive (MS-VAR) model and conclude that the stock return is capable of predicting trading volume but the evidence for trade volume predicting returns is weaker.

Another strand in this line of research focuses more on the volume-volatility relation. Ample evidence has been reported supporting a significant linkage between volume and volatility. Lee and Rui (2002) apply a VAR model to three major stock markets and find evidence of a positive feedback relationship between trading volume and return volatility. Eleanor Xu et al. (2006) employ a time-consistent vector autoregressive model to test the dynamic relationship between return volatility and trades using intraday data and report that volatility and volume are persistent and highly correlated with past volatility and volume. Hutson et al. (2008) investigate the relation between the first three moments of market returns and trading volumes in eleven international stock markets and find strong evidence of the volume-volatility relation. Chuang et al. (2012) study the major Asian stock markets and report that there is a positive bi-directional causality between trading volume and return volatility in Japan, Korea, Singapore, and Taiwan.

In spite of a large body of research on the relation among stock returns, trading volume and return volatility, surprisingly they are seldom estimated as a joint system. As mentioned by Chuang et al. (2012), stock returns, trading volume, and return volatility are jointly and simultaneously determined by the same market dynamics and a partial estimation of the system could lead to inefficient and potentially biased results. Chuang et al. apply a bivariate VAR-GARCH model to simultaneously study these relations. In light of Chen's (2012) recent work which investigates the return-volume relationship under state-dependent market condition, together with Chuang et al.'s (2012) work, this paper suggests a multivariate regime switching GARCH model to study simultaneously the empirical relation between stock return and trading volume and between stock volatility and trading volume under regime switching as a joint system.

The contribution of this paper is twofold: first I propose an asynchronous Markov regime switching generalized orthogonal GARCH (*ARSGO*) model which incorporates the framework of asynchronous switching properties (Bengoechea et al., 2006; Camacho and Perez-Quiros, 2006) with generalized orthogonal GARCH (*GO*) model (van der Weide, 2002). *ARSGO* is an extension of the regime switching generalized orthogonal GARCH model proposed by Lee (2009) such that different data series are governed by different state variables and the dependence of switching is captured by a synchronization factor.[1] Second, Chen's (2012) MS-VAR does not capture the full regime switching covariance structure of trading volume and stock returns and the dynamic relation between trading volume and stock volatility is not investigated. Chang et al.'s (2012) VAR-GARCH investigates the dynamic relation between trading volume and stock volatility but does not consider the state of market conditions. With the proposed *ARSGO*, the simultaneous estimation of stock returns, trading volume and return volatility under regime switching will be feasible. This paper attempt to investigate simultaneously the contemporaneous and causal relations between stock returns and trading volume and the causal relation between return volatility and trading volume under regime switching.

The remainder of the article is organized as follows. The asynchronous Markov regime switching generalized orthogonal GARCH (*ARSGO*) is presented in section II. Section III gives the description of recombining procedure and regime switching filtering algorithm. This is followed by discussions of data and empirical results. A conclusion ends the article.

II. Asynchronous Markov regime switching generalized orthogonal GARCH (*ARSGO*)

The proposed asynchronous Markov regime switching generalized orthogonal GARCH (*ARSGO*) model which incorporates the framework of asynchronous switching properties with *GO* GARCH such that different data series are governed by different state variables and the dependence of switching is captured by a synchronization

[1] Lee (2009) proposes a regime switching generalized orthogonal GARCH model for optimal futures hedging which restricts all data series to be governed by a common switching dynamic. *ARSGO* releases this assumption and the dependence of switching is further captured by a synchronization factor.

factor.[2] The *ARSGO* specification with application for modelling stock return and trading volume dynamics is depicted below:

Let $\mathbf{Y}_t = [R_t \quad V_t]'$ be a 2×1 vector of stock return and trading volume with conditional mean equations given by

$$R_t = \mu_{R,s_{R,t}} + \phi_{R,s_{R,t}} R_{t-1} + \phi_{RV,s_{R,t}} V_{t-1} + e_{R,s_{R,t}} , \tag{1}$$

$$V_t = \mu_{V,s_{V,t}} + \phi_{VR,s_{V,t}} R_{t-1} + \phi_{V,s_{V,t}} V_{t-1} + e_{V,s_{V,t}} , \tag{2}$$

where " $'$ " stands for transpose and $s_{R,t}$ and $s_{V,t}$ are respectively the state variables governing the regime-shifting dynamics of stock return and trading volume.

To complete the dynamic specification of the process, a new state variable $\mathbf{S}_t$ is defined and is given by

$$\mathbf{S}_t = \begin{cases} 1 & if\ s_{R,t} = 1\ and\ s_{V,t} = 1 \\ 2 & if\ s_{R,t} = 1\ and\ s_{V,t} = 2 \\ 3 & if\ s_{R,t} = 2\ and\ s_{V,t} = 1 \\ 4 & if\ s_{R,t} = 2\ and\ s_{V,t} = 2 \end{cases} . \tag{3}$$

The four-state matrix of transition probability is defined as

$$\mathbf{P}_S = \begin{bmatrix} P_{S,11} & P_{S,21} & P_{S,31} & P_{S,41} \\ P_{S,12} & P_{S,22} & P_{S,32} & P_{S,42} \\ P_{S,13} & P_{S,23} & P_{S,33} & P_{S,43} \\ P_{S,14} & P_{S,24} & P_{S,34} & P_{S,44} \end{bmatrix} , \tag{4}$$

whose columns sum to unity and $P_{S,ij} = P(\mathbf{S}_t = j \mid \mathbf{S}_{t-1} = i)$ stands for transition probability of being in state i at time $t-1$ and in state j at time t. With the definition above, the matrix form of the mean equation is given by

$$\mathbf{Y}_t = \boldsymbol{\mu}_{\mathbf{S}_t} + \boldsymbol{\varphi}_{\mathbf{S}_t} \mathbf{Y}_{t-1} + \mathbf{e}_{t,\mathbf{S}_t} , \tag{5}$$

[2] *ARSGO* is different from Otranto's (2005) multi-chain Markov switching (*MCMS*) model and Sheu and Lee's (2014) multi-chain Markov regime switching GARCH (*MCSG*) model in two aspects: Firstly, *MCMS* and *MCSG* capture the switching dependence with lagged transition variables entering into the transition probability. *ARSGO*, however, captures the dependence of switching with a synchronization factor which measures explicitly the weight of synchronous regimes. Secondly, *ARSGO* embedded van der Weide's *GO* GARCH (2002) with an asynchronous switching framework and capture the state-dependent time-varying correlation dynamic with a state-dependent mapping matrix. *MCMS* and *MCSG*, however, estimate models with a constant correlation within each regime.

where $\boldsymbol{\mu}_{\mathbf{s}_t} = \left[\mu_{R,s_{R,t}} \quad \mu_{V,s_{V,t}}\right]'$ is a 2×1 vector of state-dependent conditional means of stock return and trading volume, $\boldsymbol{\varphi}_{\mathbf{s}_t}$ is defined as

$$\begin{bmatrix} \phi_{R,s_{R,t}} & \phi_{RV,s_{R,t}} \\ \phi_{VR,s_{V,t}} & \phi_{V,s_{V,t}} \end{bmatrix}, \tag{6}$$

and $\mathbf{e}_{t,\mathbf{s}_t} = \left[e_{R,s_{R,t}} \quad e_{V,s_{R,t}}\right]' = \mathbf{Z}\left(\theta_{s_t}\right) \boldsymbol{\varepsilon}_{t,s_t}$ is a state-dependent residual vector. The state-dependent linear mapping matrix $\mathbf{Z}\left(\theta_{\mathbf{s}_t}\right)$ is given by

$$\mathbf{Z}\left(\theta_{\mathbf{s}_t}\right) = \begin{pmatrix} 1 & 0 \\ \cos\theta_{\mathbf{s}_t} & \sin\theta_{\mathbf{s}_t} \end{pmatrix}, \tag{7}$$

where $\theta_{\mathbf{s}_t}$ is a state-dependent rotation angle capture the correlation dynamic of stock return and trading volume and is governed by both state variables $s_{R,t}$ and $s_{V,t}$.

The state-dependent residual vector $\boldsymbol{\varepsilon}_{t,\mathbf{s}_t}$ assumed to be normally distributed

$$\boldsymbol{\varepsilon}_{t,\mathbf{s}_t} \mid \psi_{t-1} \sim BN\left(\mathbf{0}, \mathbf{H}_{t,\mathbf{s}_t}\right), \tag{8}$$

where BN stands for bivariate normal, ψ_{t-1} is information set available at time $t-1$ and the time-varying state-dependent covariance matrix $\mathbf{H}_{t,\mathbf{s}_t}$ is specified as

$$\mathbf{H}_{t,\mathbf{s}_t} = diag(h_{R,t,s_{R,t}}, h_{V,t,s_{V,t}}), \tag{9}$$

where each component is described by a switching GARCH model given by

The volatility dynamics of stock return and trading volume are assumed to follow a regime switching GARCH process given by

$$h_{R,t,s_{R,t}} = \gamma_{R,s_{R,t}} + \alpha_{R,s_{R,t}} e_{R,t-1}^2 + \beta_{R,s_{R,t}} h_{R,t-1} + \lambda_{R,s_{R,t}} V_{t-1}, \tag{10}$$

$$h_{V,t,s_{V,t}} = \gamma_{V,s_{V,t}} + \alpha_{V,s_{V,t}} e_{V,t-1}^2 + \beta_{V,s_{V,t}} h_{V,t-1,s_{V,t}}. \tag{11}$$

In the volatility dynamic, the lagged term of volume enters the return volatility dynamic to capture the impact of past volume on current return volatility (Chuang et al., 2012).

With the above definition, the vector $\mathbf{Y}_t$ is normally distributed with conditional mean $\boldsymbol{\mu}_{\mathrm{s}_t}$ and conditional covariance matrix $\boldsymbol{\Omega}_{t,\mathrm{s}_t}$

$$\mathbf{Y}_t \mid \psi_{t-1} \sim N\left(\boldsymbol{\mu}_{\mathrm{s}_t}, \boldsymbol{\Omega}_{t,\mathrm{s}_t}\right), \tag{12}$$

where

$$\boldsymbol{\Omega}_{t,\mathrm{s}_t} = \mathbf{Z}\left(\theta_{\mathrm{s}_t}\right) \mathbf{H}_{t,\mathrm{s}_t} \mathbf{Z}\left(\theta_{\mathrm{s}_t}\right)'. \tag{13}$$

The state-dependent correlation coefficient ρ_{t,s_t} can be shown as

$$\rho_{t,\mathrm{s}_t} = \frac{\cos\left(\theta_{\mathrm{s}_t}\right) h_{R,t,s_{R,t}}}{\sqrt{h_{R,t,s_{R,t}} \left[\cos^2\left(\theta_{\mathrm{s}_t}\right) h_{R,t,s_{R,t}} + \sin^2\left(\theta_{\mathrm{s}_t}\right) h_{V,t,s_{V,t}} \right]}}. \tag{14}$$

The contemporaneous correlation between stock return and trading volume ρ_{t,s_t} depends on both the state variables $s_{R,t}$ and $s_{V,t}$. As a consequence, there are four possible time-varying correlation dynamics under high/high, high/low, low/high and low/low return volatility and volume volatility regimes. With *ARSGO*, we can investigate the contemporaneous correlations between stock return and trading volume under different regime combinations.

III. Recombining procedure and regime switching filtering algorithm

To solve the well-known path-dependency problem (Cai, 1994; Hamilton and Susmel, 1994; Gray, 1996) due to the recursive nature and regime switching property, Gray's recombining method (Gray, 1996) is applied for residuals and volatilities:

$$e_{R,t} = R_t - E\left[R_t \mid \psi_{t-1}\right] = R_t - \left\{p_{R,t}\bar{\mu}_{R,s_{R,t}=1} + \left(1 - p_{R,t}\right)\bar{\mu}_{R,s_{R,t}=2}\right\}, \tag{15}$$

$$e_{V,t} = V_t - E\left[V_t \mid \psi_{t-1}\right] = V_t - \left\{p_{V,t}\bar{\mu}_{V,s_{V,t}=1} + \left(1 - p_{V,t}\right)\bar{\mu}_{V,s_{V,t}=2}\right\}, \tag{16}$$

where $\bar{\mu}_{R,s_{R,t}=i}$ and $\bar{\mu}_{V,s_{V,t}=i}$, $i \in \{1,2\}$ are respectively the state-dependent conditional means of stock return and trading volume and $p_{R,t} = P(s_{R,t} = 1 | \psi_{t-1})$ and $p_{V,t} = P(s_{V,t} = 1 | \psi_{t-1})$ are regime probabilities of being in state 1 at time t for state variables $s_{R,t}$ and $s_{V,t}$, respectively. The recombining process for the variance of stock return is given by

$$Var(R_t) = h_{R,t} = P(s_{R,t} = 1 | \psi_{t-1}) \left(\bar{\mu}^2_{R,s_{R,t}=1} + h_{R,t,s_{R,t}=1}\right) + \left(1 - P(s_{R,t} = 1 | \psi_{t-1})\right) \left(\bar{\mu}^2_{R,s_{R,t}=2} + h_{R,t,s_{R,t}=2}\right)$$

$$-\left[P(s_{R,t} = 1 | \psi_{t-1}) \ \bar{\mu}_{R,s_{R,t}=1} + \left(1 - P(s_{R,t} = 1 | \psi_{t-1})\right) \ \bar{\mu}_{R,s_{R,t}=2}\right]^2 , \qquad (17)$$

Analog to Lee (2009), the recombining process for the trading volume is given by

$$Var(V_t) = \sum_{1=1}^{4} \left(\bar{\mu}^2_{R,s_{R,t}} + \cos^2(\theta_{\mathbf{S}_t=i}) h_{R,t,s_{R,t}} + \sin^2(\theta_{\mathbf{S}_t=i}) h_{V,t,s_{V,t}}\right) \times P(\mathbf{S}_t = i | \psi_{t-1}) \cdot$$

$$-\left[P(s_{V,t} = 1 | \psi_{t-1}) \ \bar{\mu}_{V,s_{V,t}=1} + \left(1 - P(s_{V,t} = 1 | \psi_{t-1})\right) \ \bar{\mu}_{V,s_{V,t}=2}\right]^2 \qquad (18)$$

Since the variance of trading volume is equal to $\cos^2(\theta) h_{R,t} + \sin^2(\theta) \ h_{V,t}$, it follows that

$$h_{V,t} = \frac{Var(V_t) - \cos^2(\theta) h_{R,t_t}}{\sin^2(\theta)} , \qquad (19)$$

where the rotation coefficients θ is equal to the weighted average of θ_1, θ_2, θ_3 and θ_4 , weighted by the conditional state probability $P(\mathbf{S}_t = i | \psi_{t-1})$.

The state-dependent volatilities $h_{R,t,s_{R,t}}$ and $h_{V,t,s_{V,t}}$ are driven by different state variables $s_{R,t}$ and $s_{V,t}$, respectively and the state-dependent dynamic conditional correlation is governed by both state variables $s_{R,t}$ and $s_{V,t}$. In Lee's regime switching model *GO* GARCH (2009), all data series are governed by a single state variable, namely, $s_{R,t} = s_{V,t}$ in our application. Although the switching dynamic between stock return and trading volume might be positive correlated, the correlation should not be perfect. Following Bengoechea et al. (2006) and Camacho and Perez-Quiros (2006), in the proposed *ARSGO*, the comovement between stock return and trading volume is assumed to be δ_{VR} times the case of independent and $(1 - \delta_{VR})$ times the case of perfect dependent, where

$0 \le \delta_{VR} \le 1$. The weight δ_{VR} as a consequence measures the extent of desynchronization between stock return and trading volume.

To incorporate the asynchronous factor δ_{VR} into *ARSGO*, the following filtering algorithm is applied. If we collect all state probabilities in the vectors and define

$$\xi_{t|t-1} = \begin{bmatrix} P(\mathbf{S}_t = 1 \mid \psi_{t-1};\Theta) \\ P(\mathbf{S}_t = 2 \mid \psi_{t-1};\Theta) \\ P(\mathbf{S}_t = 3 \mid \psi_{t-1};\Theta) \\ P(\mathbf{S}_t = 4 \mid \psi_{t-1};\Theta) \end{bmatrix}, \quad \xi_{t|t} = \begin{bmatrix} P(\mathbf{S}_t = 1 \mid \psi_t;\Theta) \\ P(\mathbf{S}_t = 2 \mid \psi_t;\Theta) \\ P(\mathbf{S}_t = 3 \mid \psi_t;\Theta) \\ P(\mathbf{S}_t = 4 \mid \psi_t;\Theta) \end{bmatrix}, \tag{20}$$

where Θ is a vector of system parameters to be estimated. The updated probability vector is given by

$$\hat{\xi}_{t|t-1} = \mathbf{P}_{\mathbf{S}}\hat{\xi}_{t-1|t-1}, \tag{21}$$

where $\mathbf{P}_{\mathrm{S}}$ is the transition matrix. Because it has δ_{VR} times for the case of independent and $(1-\delta_{VR})$ times for the case of perfect dependent, $\mathbf{P}_{\mathrm{S}}$ is calculated as

$$\mathbf{P}_{\mathbf{S}} = \delta_{VR}\mathbf{P}^I + (1-\delta_{VR})\mathbf{P}^D, \tag{22}$$

Where $\mathbf{P}^I$ and $\mathbf{P}^D$ are the transition matrices in cases of independent and perfect synchronous, respectively. Substituting equation (22) into equation (21), we have

$$\begin{aligned} \hat{\xi}_{t|t-1} &= \left[\delta_{VR}\mathbf{P}^I + (1-\delta_{VR})\mathbf{P}^D\right] \hat{\xi}_{t-1|t-1} \\ &= \delta_{VR}\hat{\xi}^I_{t|t-1} + (1-\delta_{VR})\ \hat{\xi}^D_{t|t-1}, \end{aligned} \tag{23}$$

where $\hat{\xi}^I_{t|t-1} = \mathbf{P}^I\hat{\xi}_{t-1|t-1}$ and $\hat{\xi}^D_{t|t-1} = \mathbf{P}^D\hat{\xi}_{t-1|t-1}$ are the predicted probability vector for the cases of independent and perfect synchronous, respectively.

In the case when the switching dynamics of stock return and trading volume are mutually independent, elements in the transition matrix $\mathbf{P}_{\mathrm{S}}$ are calibrated as a product of those for the independent chains governing $s_{R,t}$ and $s_{V,t}$. For example,

$$P(\mathbf{S}_t = 1 \mid \mathbf{S}_{t-1} = 3) = P(s_{R,t} = 1 \mid s_{R,t-1} = 2) \times P(s_{V,t} = 1 \mid s_{V,t-1} = 1). \tag{24}$$

The forecasted probability vector in the case of mutually independent denoted as $\boldsymbol{\xi}^{I}_{t|t-1}$ is then given by:

$$\boldsymbol{\xi}^{I}_{t|t-1} = \begin{bmatrix} P(s_{R,t}=1\,|\,\psi_{t-1};\Theta)\times P(s_{V,t}=1\,|\,\psi_{t-1};\Theta) \\ P(s_{R,t}=1\,|\,\psi_{t-1};\Theta)\times P(s_{V,t}=2\,|\,\psi_{t-1};\Theta) \\ P(s_{R,t}=2\,|\,\psi_{t-1};\Theta)\times P(s_{V,t}=1\,|\,\psi_{t-1};\Theta) \\ P(s_{R,t}=2\,|\,\psi_{t-1};\Theta)\times P(s_{V,t}=2\,|\,\psi_{t-1};\Theta) \end{bmatrix}, \tag{25}$$

As in the case when both stock return and trading volume share a common pattern of regime switches, state 2 and 3 for $\mathbf{S}_t$ are excluded. In this case, the transition matrix is reduced to a 2×2 matrix and the forecasted probability vector in the case of perfect synchronization denoted as $\boldsymbol{\xi}^{D}_{t|t-1}$ is given by:

$$\boldsymbol{\xi}^{D}_{t|t-1} = \begin{bmatrix} P(s_t=1\,|\,\psi_{t-1};\Theta) \\ 0 \\ 0 \\ P(s_t=2\,|\,\psi_{t-1};\Theta) \end{bmatrix}, \tag{26}$$

where $s_t = s_{R,t} = s_{V,t}$. To estimate the transition probability matrix, the following logistic functions are applied to calculate the transition probabilities for stock return and trading volume.

$$P(s_{R,t}=1\,|\,s_{R,t-1}=1)=\frac{\exp(p_{R,0})}{1+\exp(p_{R,0})},\; P(s_{R,t}=2\,|\,s_{R,t-1}=2)=\frac{\exp(q_{R,0})}{1+\exp(q_{R,0})}, \tag{27}$$

$$P(s_{V,t}=1\,|\,s_{V,t-1}=1)=\frac{\exp(p_{V,0})}{1+\exp(p_{V,0})},\; P(s_{V,t}=2\,|\,s_{V,t-1}=2)=\frac{\exp(q_{V,0})}{1+\exp(q_{V,0})}, \tag{28}$$

where $p_{R,0}$, $q_{R,0}$, $p_{V,0}$, $q_{V,0}$ are estimated along with system parameters. The filtered probabilities are updated according to the following formula:

$$\hat{\boldsymbol{\xi}}_{t|t} = \frac{\hat{\boldsymbol{\xi}}_{t|t-1}\circ\boldsymbol{\eta}_t}{\mathbf{1}'(\hat{\boldsymbol{\xi}}_{t|t-1}\circ\boldsymbol{\eta}_t)}, \tag{29}$$

where $\mathbf{1}$ is a 2×1 vector of ones, $\circ$ denotes elements-by-elements and $\boldsymbol{\eta}_t$ is a vector of conditional density given by

$$\boldsymbol{\eta}_t = \begin{bmatrix} f(\mathbf{Y}_t \mid \mathbf{S}_t = 1) \\ f(\mathbf{Y}_t \mid \mathbf{S}_t = 2) \\ f(\mathbf{Y}_t \mid \mathbf{S}_t = 3) \\ f(\mathbf{Y}_t \mid \mathbf{S}_t = 4) \end{bmatrix}. \tag{30}$$

The density of stock return and trading volume conditional on past observations and being in regime $\mathbf{S}_t$ at time t is denoted as

$$f(\mathbf{Y}_t \mid \mathbf{S}_t = j, \psi_{t-1}; \Theta) = \frac{1}{(2\pi)\left|\mathbf{Z}(\theta_{\mathrm{S}_t})\,\mathbf{H}_{t|t-1,\mathrm{S}_t}\mathbf{Z}(\theta_{\mathrm{S}_t})'\right|^{1/2}} \times .$$

$$\exp\left\{-\frac{1}{2}\left(\mathbf{Y}_t - \bar{\boldsymbol{\mu}}^j\right)'\mathbf{Z}(\theta_{\mathrm{S}_t})\,\mathbf{H}_{t|t-1,\mathrm{S}_t}\mathbf{Z}(\theta_{\mathrm{S}_t})'^{-1}\left(\mathbf{Y}_t - \bar{\boldsymbol{\mu}}^j\right)\right\} \tag{31}$$

The mixture likelihood is the weighted average of conditional densities given by

$$f(\mathbf{Y}_t \mid \psi_{t-1}; \Theta) = \sum_{i=1}^{4} f(\mathbf{Y}_t \mid \mathbf{S}_t = j, \psi_{t-1}; \Theta) \times P(\mathbf{S}_t = j \mid \psi_{t-1}; \Theta). \tag{32}$$

The unknown parameters in *ARSGO* are $\boldsymbol{\Theta} = \{\delta_{VR},\ p_{R,0},\ q_{R,0},\ p_{V,0},\ q_{V,0},\ \mu_{R,s_{R,t}},\ \phi_{R,s_{R,t}},\ \phi_{RV,s_{R,t}},$ $\mu_{V,s_{V,t}},\ \phi_{VR,s_{V,t}},\ \phi_{V,s_{V,t}},\ \gamma_{R,s_{R,t}},\ \tau_{s_{V,t}},\ \alpha_{R,s_{R,t}},\ \beta_{R,s_{R,t}},\ \lambda_{R,s_{R,t}},\ \gamma_{V,s_{V,t}},\ \alpha_{V,s_{V,t}},\ \beta_{V,s_{V,t}},\ \theta_{\mathrm{S}_t}\}$ for $s_{R,t} \in \{1,2\}$, $s_{V,t} \in \{1,2\}$ and $\mathbf{S}_t \in \{1,2,3,4\}$, which can be estimated by maximizing the following log-likelihood function

$$LL = \sum_{t=1}^{T} \log\left[f(\mathbf{Y}_t \mid \psi_{t-1}; \Theta)\right], \tag{33}$$

where $f(\mathbf{Y}_t \mid \psi_{t-1}; \Theta)$ is defined in equation (32).

IV. Data description and empirical results

The proposed *ARSGO* is applied to weekly returns on Hang Seng index and trading volume from 1988/6/1 to 2014/12/31 and weekly returns on S&P price index and trading volume from 1986/1/8 to 2014/12/31. The data on the stock price index (p_t) and trading volume (v_t) are collected from the Datastream database. The stock returns and the percentage changes in volume are calculated as $R_t = \log(p_t / p_{t-1}) \times 100$ and $V_t = \log(v_t / v_{t-1}) \times 100$, respectively. Table 1 reports the summary statistics on stock returns and trading volume including sample mean, maximum value, minimum value, standard deviation, skewness, kurtosis and Jarque-Bera test for normality. Hang Seng index has a mean return of 0.161 which is higher than that of S&P 500 index. The mean percentage change of Hang Seng trading volume is also higher than that of S&P 500 trading volume. Higher percentage change in trading volume reflects higher volatility in trading volume that might signal a higher risk and require a higher compensated return. According to the Skewness, leptokurtosis, and significant Jarque-Bera statistics, the unconditional distributions of for index return and percentage change of volume are non-Gaussian. This justifies modeling the index return and percentage change of volume with nonlinear regime switching model.

Table II shows the *ARSGO* estimates of unknown parameters for Hang Seng returns and trading volumes and S&P 500 returns and trading volumes. In the mean equation, for the case of Hang Seng index, state 1 is the higher volatility state for both return and volume. The unconditional mean return on Hang Seng index is equal to -0.070 and 0.453 in the low and high volatility state, respectively. Higher volatility is normally associated with a market downturn and generates lower return. For the case of S&P 500 index, state 1 is the lower volatility state for return and the higher volatility state for volume. Again, the unconditional mean return is lower and equal to -0.121 in the high volatility state. Hang Seng index has unconditional mean percentage changes in volume of 0.046 and -0.089 in the high and low volatility regimes respectively and S&P 500 index has unconditional mean percentage changes in volume of 1.193 and 0.912 in the high and low volatility regimes respectively. The mean percentage changes in volume are higher in the high volatility state for both indices. This reflects the fact that higher mean percentage changes in volume causes higher trading volume volatility.

The coefficients $\phi's$ reflects the one-period own lagged or cross lagged effects. For Hang Seng index, $\phi_{R,s_{R,t}}$ is respectively equal to 0.160 and -0.085 in the high and low volatility states. It appears that in the high volatility state, the higher the return in previous period, the higher the current period return will be. Overall, Hang Seng index return shows higher persistence in the high volatility state. Similar results also found in S&P 500 index. It also has a significant higher $\phi_{R,s_{R,t}}$ of 0.185 in the high volatility state (state 2) and shows higher return persistence in the high volatility state. $\phi_{V,s_{V,t}}$ shows the effect of lagged percentage change in trading volume on its current value. For Hang Seng index, $\phi_{V,s_{V,t}}$ is equal to 0.314 in the high volatility state and -0.559 in the low volatility state. The higher percentage change in trading volume last period results in a higher percentage change in trading volume this period in the high volatility state but results in a lower percentage change in trading volume this period in the lower volatility state. S&P 500 index also exhibits similar property. In general, percentage change in trading volume has higher return persistence in the high volatility state for both indices. The impact of one-period lagged percentage change in volume on current return is reflected in the sign and magnitude of $\phi_{RV,s_{R,t}}$. In the high volatility state, higher positive percentage change in volume last period causes higher return for Hang Seng index and lower return for S&P 500 index. The relationship between lagged trading volume and current return, however, is insignificant for both indices in the low volatility state. $\phi_{VR,s_{V,t}}$ shows the impact of one-period lagged return on current trading volume. The lagged index return has a significant negative impact on current trading volume for both indices in the high volatility state signals that investors are more hesitate in actively trading when stock rally during market turmoil. The index return, however, has a significant positive impact on current trading volume for both indices in the low volatility state. This shows that investors are more actively in trading when return increases in the relatively lower market volatility state.

All $\gamma's$ in the high volatility state is higher than that in the low volatility state in the variance equation shown in Table II. The γ estimates from *ARSGO* for Hang Seng and S&P 500 return in the high volatility state are 0.834 and 0.524, respectively. The estimates in the low volatility state for returns, however, are close to zero for both indices. The γ estimates from *ARSGO* for Hang Seng and S&P 500 percentage changes in trading volume are respectively equal to 0.730 and 0.806 in the high volatility state and are respectively equal to 0.325 and 0.424 in the low volatility state. Higher γ indicates a higher steady state volatility For a given α and β. The term $\alpha+\beta$ in the volatility equation measures the volatility persistence. $\alpha+\beta$ in the high and low volatility states are respectively equal to 1 and 0.894 for Hang Seng stock return and the $\alpha+\beta$ in the high and low volatility states are respectively equal to 1 and 0.789 for S&P 500 stock return. Similar results also found for the trading volume. In general, the volatility persistence tends to be more pronounced in higher return or trading volume volatility regime. $\lambda_{R,s_{R,t}}$ indicates the state-dependent impact of lagged trading volume on current return volatility. All estimated coefficients are close to zero and insignificant indicating that the lagged percentage changes in trading volume does not have an impact on current return volatility. The state-dependent volatilities of Hang Seng index return and percentage changes in trading volume are respectively shown in figure 1 and figure 2 and the state-dependent volatilities of S&P 500 index return and percentage changes in trading volume are respectively shown in figure 4 and figure 5.

The contemporaneous correlations dynamic of stock return and trading volume is a function of the state-dependent rotation angle θ_{S_t} derived in equation (14). Because index return and percentage changes in volume are govern by different state variables $s_{R,t}$ and $s_{V,t}$ following individually a first-order two-state Markov chain, there are four possible correlation regimes in each point of time. The state-dependent rotation angle between index return and percentage changes in volume for Hang Seng index are equal to 0.728 and 0.194 when both of them are simultaneously in the high and low volatility states, respectively. This can be converted into average correlations of 0.053 and 0.106 when both of them are respectively in the high and low volatility states based on equation (14). The state-dependent rotation angle between index return and percentage changes in volume for Hang Seng index are equal to 0.258 and 0.049 when return is in the high volatility state and volume is in the low volatility state and vice versa, respectively. The corresponding average correlations are 0.188 and 0.627, respectively. Return and volume have higher correlation when they are in different volatility regimes. For S&P 500 index return, however, the correlation is higher when index return and percentage changes in volume are in the same volatility regimes than in different volatility regime. The average correlations are respectively equal to 0.140 and 0.124 when both index return and percentage changes in volume are in the high and low volatility regimes and the average correlations are respectively equal to 0.088 and -0.01 when index return and percentage changes in volume are respectively in the high and low volatility regime and vice versa. Generally speaking, the average correlations in all regime combinations between index return and percentage changes in volume for Hang Seng index are higher than that of S&P 500 index. The comovement between stock return and trading volume is assumed to be δ_{VR} times the case of independent. It is shown that the weight δ_{VR} is equal to 0.194 and 1 for Hang Seng and S&P 500 indices, respectively. The extent of desynchronization between stock return and trading volume is much higher in S&P 500 index than in Hang Seng index. This is consistent with the results that correlation between return and volume for Hang Seng index are higher than that of S&P 500 index in all regime combinations. The state-dependent correlations of Hang Seng index and S&P 500 index are shown in figure 3 and figure 6, respectively.

V. CONCLUSIONS

The focus of this article has been investigating simultaneously the contemporaneous and causal relations between stock returns and trading volume and the causal relation between return volatility and trading volume under multiple-state-variable regime switching. An asynchronous Markov regime switching generalized orthogonal GARCH (*ARSGO*) model is proposed to make feasible of one-step estimation of stock returns, return volatility and trading volume under regime switching. *ARSGO* is an extension of *GO* GARCH model such that different financial variables are governed by different state variables with the dependence of switching captured by a synchronization factor.

The suggested *ARSGO* is applied to Hang Seng and S&P 500 index returns and percentage changes in trading volumes. Empirical results show that unconditional mean returns on both Hang Seng and S&P 500 indices are lower in the higher volatility state. Higher volatility is normally associated with a market downturn and creates lower return. The mean percentage changes in volume, however, are higher in the high volatility state for both indices. This reflects the fact that higher mean percentage changes in volume might cause higher trading volume volatility. Results also show that the higher the return in previous period, the higher the current return will be in the high volatility state for both Hang Seng and S&P 500 indices. Overall, the index returns exhibit higher return persistence in the high volatility state. The higher percentage change in trading volume last period results in a higher percentage change in trading volume this period in the high volatility state but results in a lower percentage change in trading volume this period in the lower volatility state for both Hang Seng and S&P 500 indices. In general, percentage change in trading volume has higher mean persistence in the high volatility state for both indices. In the high volatility state, higher one-period lagged percentage change in volume causes higher current return for Hang Seng index but lower current return for S&P 500 index. The relationship between lagged trading volume and current return, however, is insignificant in the low volatility state for both indices. The lagged index return has a significant negative impact and a significant positive impact on current trading volume in the high and low volatility state respectively for both indices.

Base on the estimation results of volatility equation, all $\gamma's$ in the high volatility state is higher than that in the low volatility state. The volatility persistence tends to be higher in the higher return or trading volume volatility regimes. It is also found that no significant impact of lagged percentage changes in trading volume on current return volatility. As for the contemporaneous correlations, stock return and percentage changes in volume have higher correlation when they are in different volatility regimes for Hang Seng index but have higher correlation when both of them are in the same volatility regimes for S&P 500 index. Generally speaking, the average correlations in all regime combinations between index return and percentage changes in volume for Hang Seng index are higher than that of S&P 500 index and as a consequence the extent of desynchronization between stock return and trading volume is much higher in S&P 500 index than in Hang Seng index.

Table I
Summary Statistics for Hang Seng and S&P 500 Stock Return and Volume

	Hang Seng		*S&P 500*	
	Index	Volume	Index	Volume
Mean	13332.820	3425262	7780.494	487393
Maximum	31352.580	36482530	15657.630	1976051
Minimum	2224.660	44620	2758.800	14959
Std. Dev.	6667.924	4185569	3809.946	391470
Skewness	0.129	1.639	0.326	0.659
Kurtosis	2.065	6.759	1.709	2.531
Jarque-Bera	54.450***	1438.243***	132.087***	123.728***
	Return	Percentage rate	Return	Percentage rate
Mean	0.161	0.340	0.107	0.239
Maximum	15.557	189.176	7.833	130.328
Minimum	-20.977	-147.664	-15.263	-154.934
Std. Dev.	3.404	40.526	2.145	29.500
Skewness	-0.586	0.328	-0.961	-0.535
Kurtosis	6.014	4.438	7.933	8.185
Jarque-Bera	604.117***	144.424***	1769.147***	1769.475***

*** indicates significance at the 1% level and returns are calculated as the differences in the logarithm of prices multiplied by 100.

Table II
Estimates of Unknown Parameters for Hang Seng and S&P 500 Stock Return and Volume

	Hang Seng	*S&P 500*		*Hang Seng*	*S&P 500*
	Mean Equation			*Variance Equation*	
μ_{r1} [2]	-0.070	0.441	γ_{R1}	0.834	0.000
	(0.086)	(0.091)***[1]		(0.489)*	(0.024)
μ_{r2} [2]	0.453	-0.121	γ_{R2}	0.000	0.524
	(0.147)***[1]	(0.115)		0.008)	(0.226)**
μ_{v1}	0.046	1.193	γ_{V1}	0.730	0.806
	(0.062)	(0.381)***		(1.438)	(1.168)
μ_{v2}	-0.089	0.912	γ_{V2}	0.325	0.424
	(0.096)	(0.936)		(0.848)	(0.270)
ϕ_{r1}	0.160	-0.126	α_{R1}	0.119	0.278
	(0.058)***	(0.049)**		(0.048)**	(0.053)***
ϕ_{r2}	-0.085	0.185	α_{R2}	0.126	0.066
	(0.089)	(0.061)***		(0.076)*	(0.035)*
ϕ_{rv1}	0.008	0.002	α_{V1}	0.000	0.000
	(0.004)*	(0.003)		(0.045)	(0.004)
ϕ_{rv2}	-0.004	-0.006	α_{V2}	0.207	0.288
	(0.003)	(0.003)**		(0.114)*	(0.099)***
ϕ_{vr1}	-0.196	-2.093	β_{R1}	0.881	0.502
	(0.076)***	(1.110)*		(0.088)***	(0.090)***
ϕ_{vr2}	0.624	0.489	β_{R2}	0.768	0.931
	(0.213)***	(0.209)**		(0.116)***	(0.090)***
ϕ_{v1}	0.314	0.673	β_{V1}	1.000	1.000
	(0.007)***	(0.034)***		(0.076)***	(0.286)***
ϕ_{v2}	-0.559	-0.450	β_{V2}	0.793	0.551
	(0.037)***	(0.021)***		(0.091)***	(0.061)***
	Correlation Equation		λ_{R1}	0.000	0.000
ρ_{11}	0.728	1.644		(0.024)	(0.006)
	(0.173)***	(0.222)***	λ_{R2}	0.000	0.003
ρ_{12}	0.258	0.185		(0.010)	(0.005)
	(0.088)***	(0.044)***			
ρ_{21}	0.049	0.153			
	(0.095)	(0.031)***			
ρ_{22}	0.408	0.319			
	(0.089)***	(0.078)***			
δ_{VR}	0.194	1.000			
	(0.110)*	(0.008)***			
LL [3]	-10465.81	-10103.81			

1. Figures in parentheses are standard errors and *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.
2. The subscripts 1 and 2 stand for state 1 and 2, respectively. State 1 is the higher volatility state for both return and percentage change in volume for Hang Seng index. As for S&P 500 index, state 1 is the lower volatility state and the higher volatility state for return and percentage change in volume, respectively.
3. *LL* stands for log likelihood.

Figures

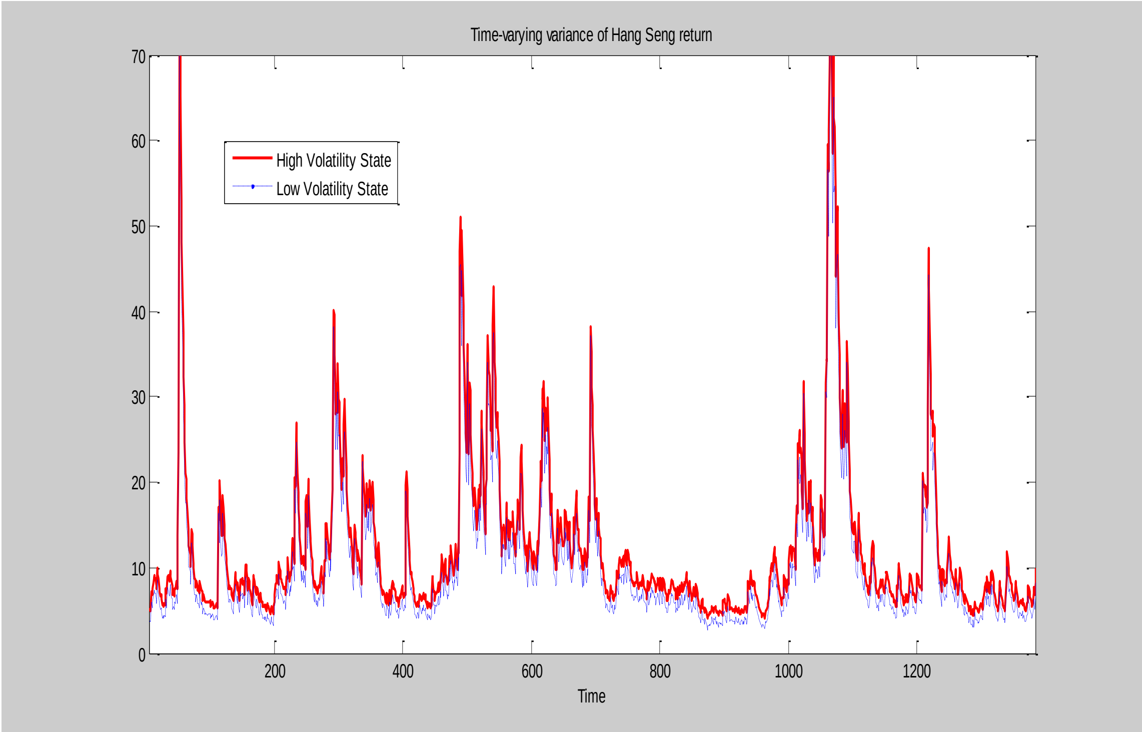

Figure 1 Regime switching variance of Hang Seng return

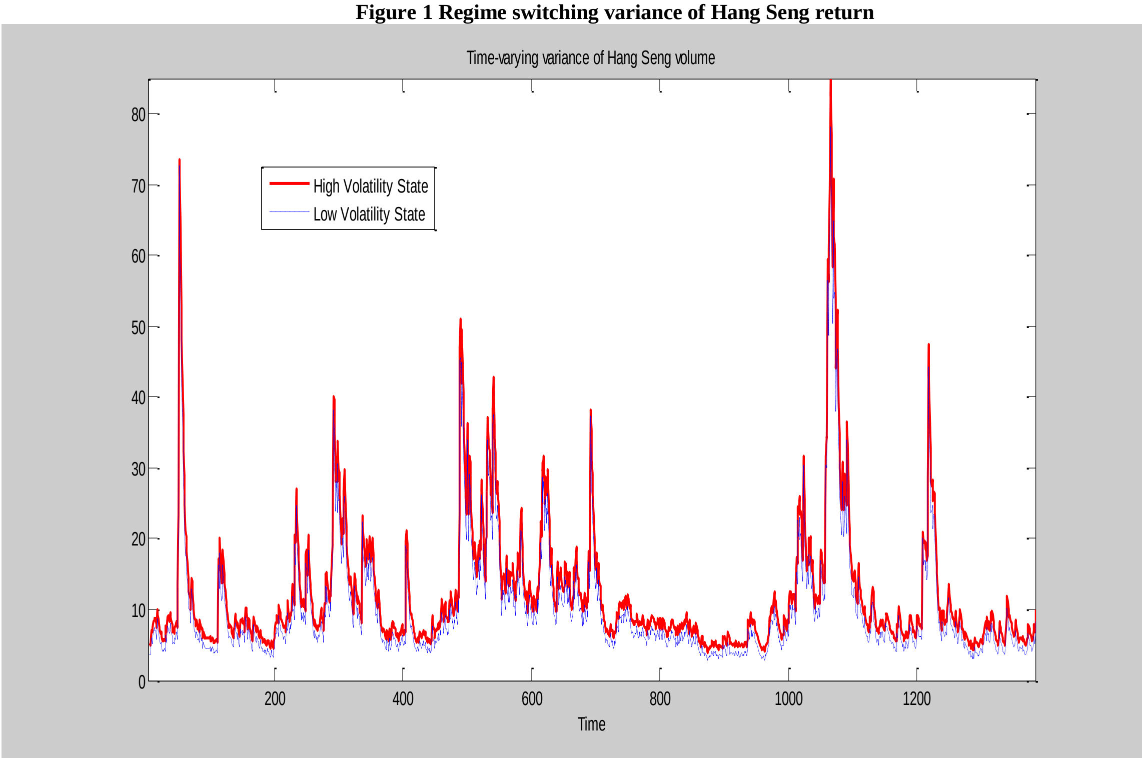

Figure 2 Regime switching variance of Hang Seng volume

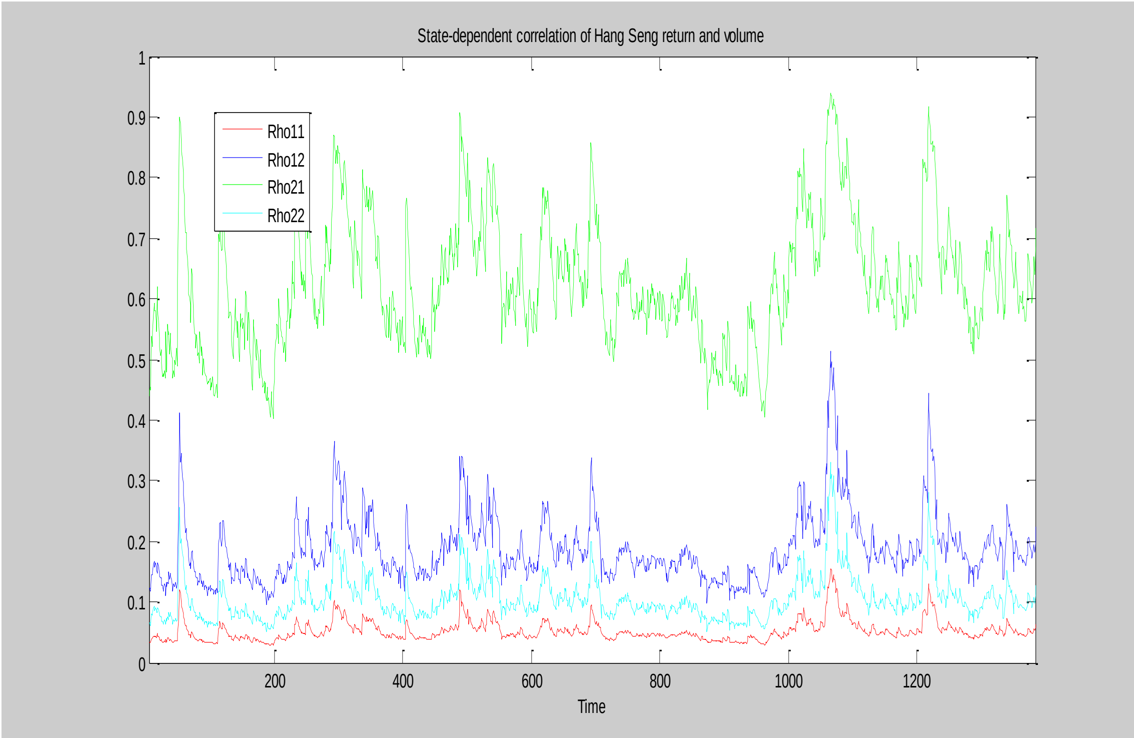

Figure 3 Regime switching correlations of Hang Seng return and volume

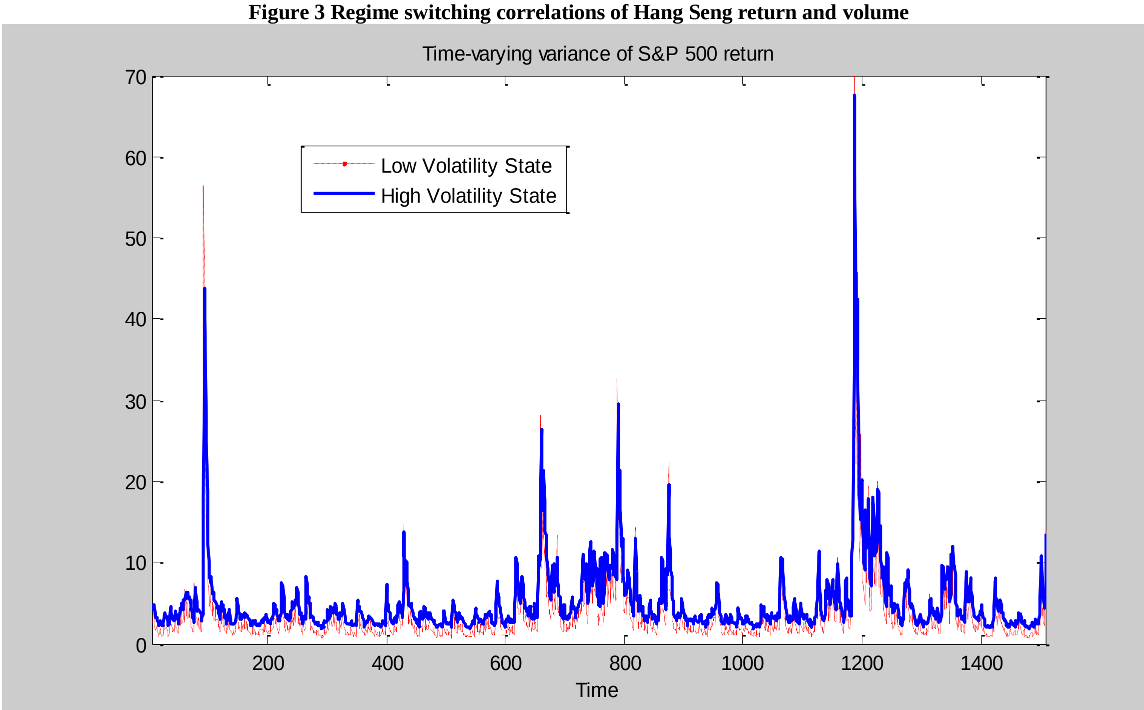

Figure 4 Regime switching variance of S&P 500 return

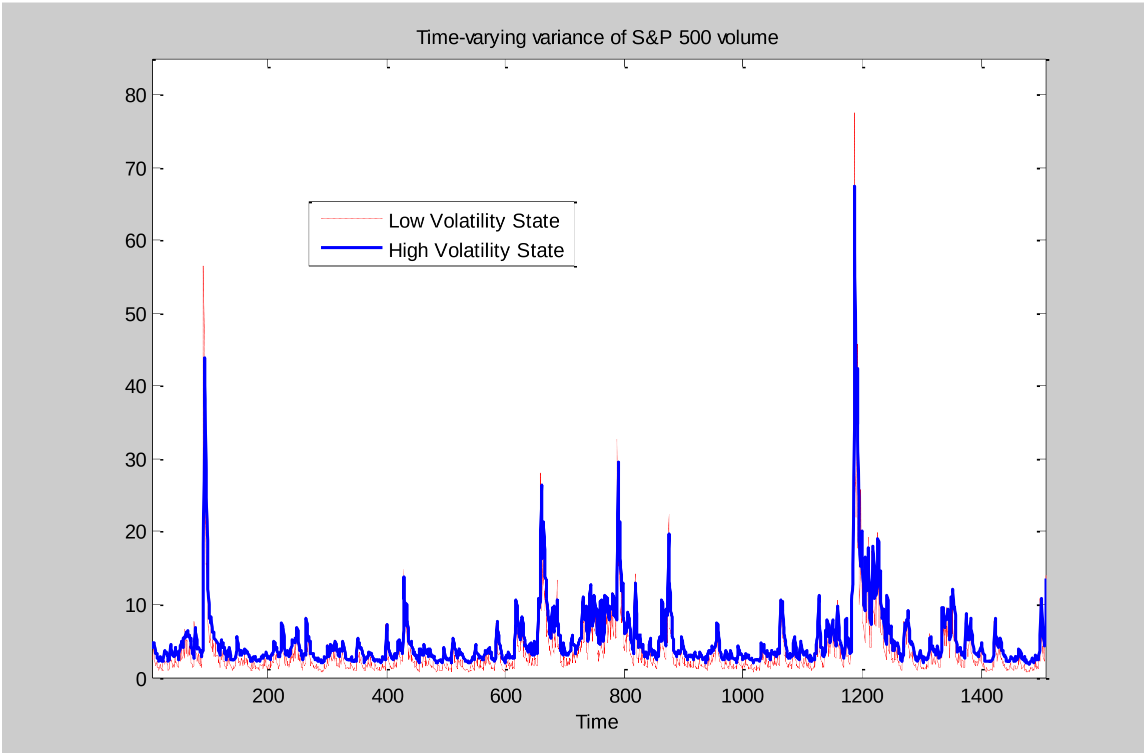

Figure 5 Regime switching variance of S&P 500 volume

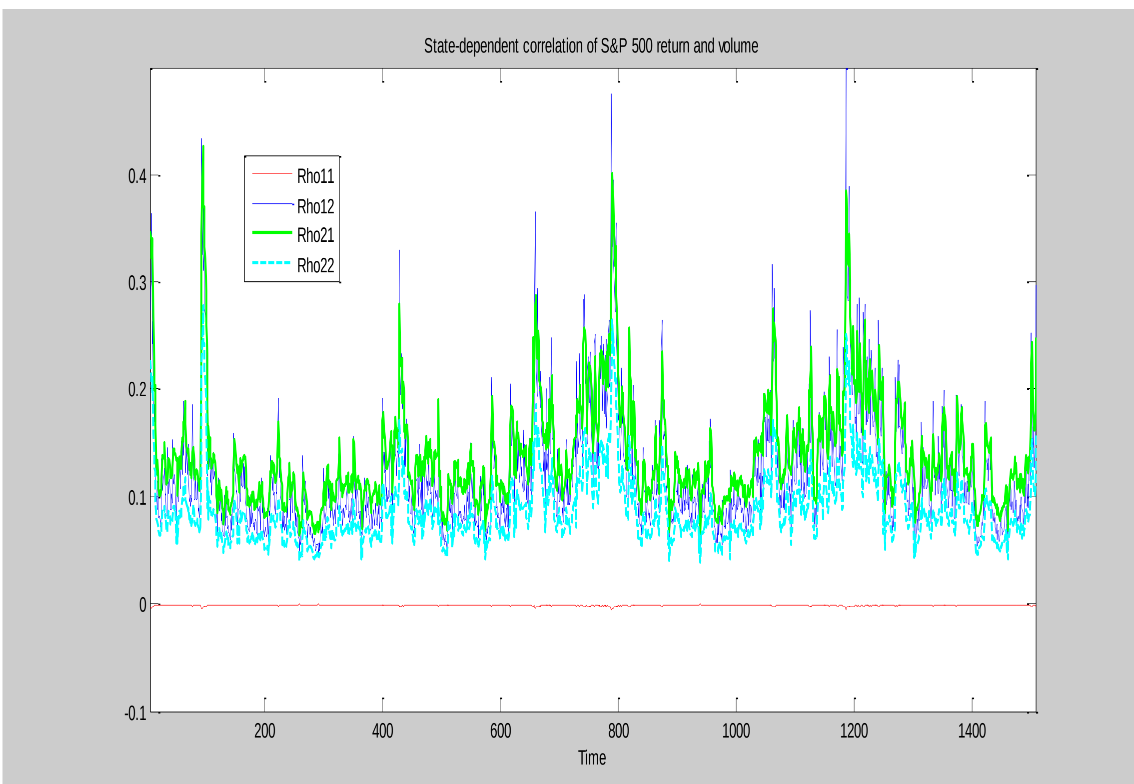

Figure 6 Regime switching correlations of S&P 500 return and volume

Appendix A. Summary of estimation procedure for *ARSGO*

To complete the likelihood function in (33) for the proposed asynchronous regime switching generalized orthogonal GARCH model (*ARSGO*), the filtering algorithm is summarized below:

(i) Given the filtered probabilities $\hat{\boldsymbol{\xi}}_{t-1|t-1}$, projects the state probabilities

$$\hat{\boldsymbol{\xi}}_{t|t-1} = \left[\delta_{VR}\mathbf{P}^{I} + (1-\delta_{VR})\mathbf{P}^{D}\right]\hat{\boldsymbol{\xi}}_{t-1|t-1}, \tag{23}$$

where $\mathbf{P}^{I}$ and $\mathbf{P}^{D}$ are respectively the transition variable in cases of independent and perfect synchronous.

(ii) Evaluate the regime dependent likelihood

$$f\left(\mathbf{Y}_t \mid \mathbf{S}_t = j, \psi_{t-1};\Theta\right) = \frac{1}{(2\pi)\left|\mathbf{Z}(\theta_{\mathrm{S}_t})\,\mathbf{H}_{t|t-1,\mathrm{S}_t}\mathbf{Z}(\theta_{\mathrm{S}_t})'\right|^{1/2}} \times .$$

$$\exp\left\{-\frac{1}{2}\left(\mathbf{Y}_t - \bar{\boldsymbol{\mu}}^{j}\right)'\mathbf{Z}(\theta_{\mathrm{S}_t})\,\mathbf{H}_{t|t-1,\mathrm{S}_t}\mathbf{Z}(\theta_{\mathrm{S}_t})'^{-1}\left(\mathbf{Y}_t - \bar{\boldsymbol{\mu}}^{j}\right)\right\} \tag{31}$$

where the mapping matrix $\mathbf{Z}(\theta_{\mathrm{S}_t})$ and volatility elements $\mathbf{H}_{t|t-1,\mathrm{S}_t}$ are defined in equations (7), (9) and (10).

(iii) Evaluate the mixture likelihood

$$f\left(\mathbf{Y}_t \mid \psi_{t-1};\Theta\right) = \sum_{i=1}^{4} f\left(\mathbf{Y}_t \mid \mathbf{S}_t = j, \psi_{t-1};\Theta\right) \times P\left(\mathbf{S}_t = j \mid \psi_{t-1};\Theta\right). \tag{32}$$

where the projected probabilities $P(\mathbf{S}_t = j \mid \psi_{t-1};\Theta)$ are estimated in step (i).

(iv) Update the joint probabilities

$$\hat{\boldsymbol{\xi}}_{t|t} = \frac{\hat{\boldsymbol{\xi}}_{t|t-1} \circ \boldsymbol{\eta}_t}{\mathbf{1}'\left(\hat{\boldsymbol{\xi}}_{t|t-1} \circ \boldsymbol{\eta}_t\right)} \tag{29}$$

Where $\mathbf{1}$ is a 2×1 vector of ones, $\circ$ denotes elements-by-elements and $\boldsymbol{\eta}_t$ is a vector of conditional density defined in equation (30).

(v) Recombining

Apply equations (15)-(19) to recombine residuals, volatilities for next period volatility projection and the rotation coefficients θ is a weighted average of θ_1, θ_2, θ_3 and θ_4 using projected state probabilities.

(vi) Iterate (i) to (v) until the end of the sample and the likelihood is obtained as a by-product of this filter given by

$$L(\Theta) = \sum_{t=1}^{T} \log\left(\mathbf{1}'\left(\hat{\boldsymbol{\xi}}_{t|t-1} \circ \boldsymbol{\eta}_t\right)\right) \tag{33}$$

The vector of steady state probabilities are used as initial regime probabilities to initialize the filtering algorithm which is given by

$$\boldsymbol{\pi} = \begin{bmatrix} P(\mathbf{S}_0 = 1) \\ P(\mathbf{S}_0 = 2) \\ P(\mathbf{S}_0 = 3) \\ P(\mathbf{S}_0 = 4) \end{bmatrix},$$

which is the solution of the system of equations $\mathbf{P}\boldsymbol{\pi} = \boldsymbol{\pi}$ and $\mathbf{1}'\boldsymbol{\pi} = \mathbf{1}$. The solution can be derived as

$\boldsymbol{\pi} = (\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\mathbf{v}_{2^n+1}$, where $\mathbf{A} = \begin{bmatrix} \mathbf{I}_{2^n} - \mathbf{P} \\ \mathbf{1}' \end{bmatrix}$, $\mathbf{v}_{2^n+1} = \begin{bmatrix} \mathbf{0}_{2^n} \\ 1 \end{bmatrix}$, n is the number of state variables, $\mathbf{I}_{2^n}$ is a $2^n \times 2^n$ identity matrix and $\mathbf{0}_{2^n}$ is a $2^n \times 1$ zero vector.

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