TWO-DIMENSIONAL BAYESIAN PERSUASION

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Abstract

We model a setup where a candidate attempts to convince multiple voters of her quality. In doing so, the candidate uses a Bayesian Persuasion style informational design mechanism. When di erent voters are convinced by di erent topics with potential correlations, the optimal mecha-nism takes on a form di erent from the standard one in the original Bayesian Persuasion paper. We show that public signals reduce the ability of the candidate to extract surplus, and improve the welfare of the voters relative to private signals. Therefore, our results add to the growing literature clarifying the bene ts of transparency in the political sphere.

1 Introduction

The transparency around political decision-making, or the lack there-of, has become a major talking point in recent decades. For instance, at the beginning of the Covid-19 pandemic, political decision-making in the senate became a key issue. Amid the impending economic recession that seemed destined to have deleterious impacts on almost every American, several senators made suspiciously timed purchases and sales in the stock market. In doing so, they likely abused privileged, publicly undisclosed information acquired as a result of being a political elite (Benner and Fandos, 2020).

Transparency is intuitively appealing because it improves the alignment between politicians' incentives and the goals of the electorate. Conversely, information asymmetry is a problem, especially in the modern political landscape. In various political contexts, a candidate may possess information that voters may not know. Such information is also only revealed to voters at the discretion of the candidate. A political candidate can use this to his or her bene t through selectively revealing information, leading voters to believe that the candidate stands for a voter's favored political state, or that the candidate is the best choice for that voter.

We model this issue of information asymmetry through an information signaling game based on the Bayesian Persuasion setup. Each voter cares about a unique issue, and the candidate attempts to convince a majority of voters. There are two key di erences between our model and the standard Bayesian Persuasion setup. First, there are multiple receivers (voters), and so the candidate needs to trade o the release of information when attempting to convince voters. Second, the candidate does not wish to persuade all the voters. Instead, the candidate wishes to persuade a majority. Importantly, the issues that voters care about may be correlated in arbitrary manners. We place no restrictions upon the shapes, positive or negative, of the correlation structure between di erent voter's issues. While the candidate understands the correlation, she cannot change it.

Resultantly, one key feature of the solution, is that the candidate cares a great deal about the correlation structure between voters. For instance, if two voters care about positively correlated issues, then in e ect the candidate gets a \two-for-one." Convincing one of the voters goes a long way towards convincing the other as well. Furthermore, an implication of the ndings suggest that the candidate needs to be strategic about the process by which she disseminates information. There are clear distinctions in the probabilities with which voters can be convinced to vote, and, by extension, the probability with which the candidates will be elected. The optimal mechanism maximizes this, through focusing on convincing voters in states where their preferences are correlated. The candidate will be required to have a thorough understanding of the values and preferences of her constituents in order to maximize her probability of election. For instance, segregating potential voters into demographics may provide the candidate with a rough estimate of individual voter preferences. The optimal mechanism as determined through this paper uncovers the means by which the candidate can maximize her probability of election. Another implication points towards di erences in the behavior of the candidate in a political setting involving public information. The candidate will be forced to rely on how voters update their beliefs upon having seen all the information, as opposed to providing targeted messages speci c to each voter. This constrains her ability to convince voters signi cantly, to their gain, and her detriment.

2 Literature Review

Our model is built directly upon the foundations of the original Bayesian Persuasion model, found in Kamenica and Gentzkow (2011). The original model considers the problem where one sender aims to design an information structure in order to maximize the probability that the receiver selects a certain action. Later, the same authors consider an extension to the problem with multiple senders in Gentzkow and Kamenica (2017) and Gentzkow and Kamenica (2016).

The implications of these papers di er crucially from our model in that multiple senders create an incentive for competition between senders, leading to an increase in information revelation. Wang (2013) considers the reverse, a persuasion setting with multiple receivers. The authors also compare public persuasion to private persuasion, focusing on instances where each receiver acts as if they are pivotal. We treat the reverse case, as in our model, every receiver behaves sincerely. Our rationale is the following: in the setting of public voting, sincere voting seems more realistic. It is di cult to justify pivotal voting in large elections, especially when the cost of voting is non-zero. Should voters condition on pivotality rather than sincerity, they'd simply opt to not vote given that their probability of pivotality is near zero.

Alonso and C^amara (2016) consider an environment similar to ours, where a politician designs an experiment in order to convince a su cient pool of voters to vote for her. The focus of their paper is upon the negative externalities voters impose upon one another through their voting patterns. In particular, they carefully examine the welfare implications generated by sel sh voting patterns when exploited by a politician. A key di erence in our paper is that we focus upon the shape of the optimal mechanism utilized by the candidate to convince voters.

3 Model

Consider a candidate who is running for o ce. There are N > 1 single-issue voters. In order to be elected, the candidate must convince a strict majority to vote for her. Each voter i 2 N has a binary action space f0; 1g, representing not voting and voting. Voter i's utility function is given by u_i : $_i$ f0; 1g ! R, where $_i$ = f0; 1g is a binary state space unique to each worker. The voter prefers to vote if the state is 1, and otherwise prefers not to vote. Phrased di erently, each voter i cares about the candidate's position on an issue $!_i$ and will vote for the candidate only if the probability the candidate agrees with the voter (the probability that $!_i$ = 1) is at least $_i$. The joint probability distribution of = $_i$ $_i$ is given by F (!). Voters share a common prior over with the candidate, F (!).

We assume that has full support, namely 8! 2 ; F (!) > 0.

The candidate's utility V : 2^N ! R depends directly upon the number of voters that voted for her. Throughout most of this paper, we will focus on strict majority based elections, and set V (x) = 0 for x N=2 or V (x) = 1 for x > N=2. Then, her evaluation of a policy is simply the expectation that at least half of the voters vote for her.

The candidate reveals information about her position selectively. That is, she commits to an information structure in a manner similar to Kamenica and Gentzkow (2011) where she uses a signaling policy that maps probabilistically from the state to messages to the receivers. Formally, the candidate chooses a set of possible messages and a platform V : ! .

When combined with the prior, p(!), this induces a probability distribution over . For ! 2 , we use P (!) to denote the expected signal distribution given !. This in turn determines the probability that she receives a majority of votes and is elected. By a standard appeal to the revelation principle, we restrict our attention to direct mechanisms, and focus on the posteriors induced by those mechanisms.

We will use the following notation to denote the ex-post probability of W_i given a mechanism

as inferred by a voter:[1]

$$P_i( ) \quad \underline{=(1;v);v2f0;1g^{N\ 1}\ ()F\ ()} \quad (1)$$

$_{v2f0;1g}N$ [(v)F (v)]

Similarly, let V F denote the standard dot product, namely:

$$V \; F \quad {}^{X} V()F() \qquad (2)$$

$$2f0;1g^{N}$$

Where appropriate, we will denote the restricted dot product by:

$$(V \; F \, j_{A} \quad V(\,)F(\,) \qquad (3)$$

X

Using the above notation $P_i(\,)$ can be rewritten as:

$$P_i(\,) = \frac{(\;\; F \, j_{1} \;\; {}_{f0;1g}N \;\; 1}{F} \qquad (4)$$

The candidate aims to maximize this probability, that is the candidate solves the following problem:

V = arg max V F s.t

V

$8i \; P_i(V\,) \qquad {}_{i}$

4 Simple Example

In this section, we provide a simple example to provide the intuition for our key results. A candidate wishes to convince two voters to vote for her. Voter 1 cares about whether the state is \Left" or \Right", while voter 2 cares about \Conservative" and \Socialist." The joint distributions over the states are

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[1] Let 1 be the identity vector and e_i be the ith elementary vector in R^N

P(LC) = 1=8

P(LS) = 3=8

P(RS) = 1=8

P(RC) = 3=8

Voter one will vote for the candidate if the probability of L is at least 4=7 and voter two will vote for the candidate if the probability of C is at least 3=5. The candidate commits to sending certain messages conditional on the actual state. What is the best she can do if she aims to maximize the probability that both simultaneously vote for her?

First, consider the strategy she would use to convince only voter 1 alone. Disregarding the other voter, the candidate now has to convince this voter that he or she stands for the favored political state of the given voter. No compromise is needed, as this voter has unique political interests that do not clash with any other voters. Therefore, if voter 1 was the voter in question, the candidate would present himself or herself as a candidate who staunchly supports political state L. Likewise, the candidate will perform the same actions if voter 2 was the voter at hand, corresponding to the voter's political interests.

Taking the above as given, this implies that if the candidate always tells voter 1 when the state is L, she can also lie when the state is R a total of 3=8 of the time. Then, the optimal strategy for convincing voter 1, can be shown as the following:

$$P(S_l j L) = 1$$

$$P(S_l j RC) = \frac{3=8}{3=8} = 1$$

To see why this is the case, consider the following. First, since the candidate wishes to convey to voter 1 that they stand for \L" as much of the time as they are able to, the candidate will send a signal \L" de nitively if the given condition is \L". Next, in state \RC", the candidate will misinform the voter and send a signal of

\L", but not so much that the voter's minimum criteria of \L" required to vote for this candidate is violated. However, conditional on sending L when the state is RC, a signal of \L" in state RS would place the voter's perceived probability of L below voter 1's required threshold, therefore it must be the case that P (s_ljRS) = 0.

The optimal strategy for voter 2, who requires at least a 3/5 probability of \C" to be convinced to vote for a candidate, can be shown as the following:

$$P\ (S_c jC) = 1$$

$$P\ (S_C jLS) = \frac{1=3}{3=8} = 8=9$$

In contrast to voter 1, voter 2 is seeking a certain minimum value of \C" in a candidate in order to cast a vote. Therefore, when the given probability of a political state is \C", the candidate will de nitively send voter 2 the signal that he or she stands for \C". While in \LS", no probability of "C" exists, the candidate is able to meet the required amount of "C" even if he or she misinforms the voter 8/9 of the time. By extension, there is no scenario in which a candidate sends the signal \C" given \RS", as that would place the candidate below the minimum value of "C" imposed by voter 2.

Then, the strategy is as follows: the candidate sends two signals, the rst being s_l and s_r and the second being s_s and s_c. If voter 1 sees s_l, he votes. Similarly, if voter 2 sees s_s, he votes. The probability that L and S are both simultaneously sent under this strategy is 5=6. However, when voter 1 observes s_s, he should update his posterior, which decreases the probability that the state is L to below 4=7. Then, voter 1 would not vote for the candidate. Therefore, this strategy only works if the candidate is able to send private signals. In that event, she can achieve the maximum. However, as we will show in the succeeding section, the optimal mechanism under public signals is quite di erent.

4.1 Optimal Mechanism

In this section, we proceed by determining the optimal mechanism in general when there are two voters. The following simple mathematical fact will prove useful below:

Remark 1. If $x > 0$ and 1 n=d, then:

x + n n

x + d d

Proof.

x + n n

x + d d

d(x + n) n(x + d)

dx + dn nx + nd

dx xn

d n

1 n=d

The simple algebra above implies that $\frac{x+n}{x+d}$ ____ $\frac{n}{d}$ if and only if 1 n=d. □

Lemma 1. If V is an optimal mechanism, then V (1) = 1.

Proof. In order to determine the optimal mechanism, we begin by considering V (1). First,

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suppose V (1) = 1 for some solution V to the candidate's problem. Let V^0 be an alternate

0 6 0

mechanism where V (1) = 1 and for any = 1; V ( ) = V ( ).

We will show that V^0 satis es all voters' incentive constraints. Since V satis ed the rst voter's incentive constraint, IC_1, by assumption, it must be the case that P_1(V) . We then need to prove that $P_1(V^0)$ is greater than $P_1(V)$. Let $n_1(V)$ and $d_1(V)$ denote the numerator and denominator of $P_1(V)$ respectively. By substituting $V(\ )$ for $V^0(\ )$ whenever $(\ )$ 6= 1, we know that:

$P_1(V^0) =$ $^{(V\ 0\ \ F\ j}$ 2ff1;1g;f1;0gg

V $\quad 0\quad$ F

(1 $\quad$ V (1))F (1) + (V $\quad ^{F\ j}$ 2A $\;$ 1 $\;$ f0;1g$^{N\ \ 1}$

= ———————————————

(1 $\qquad$ V (1))F (1) + V $\;$ F

(1 $\quad$ V (1))F (1) + n_1 (V)

= ———————————————

(1 $\quad$ V (1))F (1) + d_1 (V)

(5)

(6)

(7)

Then by Remark 1, $P_1(V^0)$ is greater than or equal to $P_1(V)$, therefore V^0 satis es IC_1. By symmetric math, V^0 satis es all incentive constraints corresponding to other voters as well.

Next, we show that $V^0 F$ is greater than $V F$, and therefore the candidate prefers V^0 to V.

0 0 0

The net gain from V , (V V) F is equal to (1 V (1))F (1), since V is V except at 1. Since

6 0

V (1) = 1, this value is positive, therefore V improves on V in a non-trivial manner.

□

To return our focus to the two voter case. Lemma 1 states a mechanism can only be optimal if V (1; 1) = 1. We proved that since both voter 1 and voter 2 prefer the political states present in

V (1; 1), it holds true that are more willing to vote if V (1; 1) is increased. As mentioned before, Lemma 1 revolves around the condition that if V (1; 1) is not equal to 1, the mechanism can be strictly improved by increasing the total probability of voting without violating any individual voter's incentive constraint. Therefore, we can conclude that any optimal mechanism V must have V (1; 1) = 1.

Lemma 2. If V is an optimal mechanism and V (0; 0) > 0, there exists an optimal mechanism V^0 such that $V^0(1; 1) = V^0(1; 0) = V^0(0; 1) = 1$.

Proof. Let V be an optimal mechanism and therefore incentive compatible, such that V (0; 0) > 0. We begin by de ning a new mechanism V^0 as follows. Based on the already proven rst

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lemma, since V is optimal, V (1; 1) = 1. By extension of the rst proposition, V (1) = 1 is optimal as well.

Next, we set the value of $V^0(1; 0)$ equal to the sum of V (1; 0) and the \excess weight" from

V (0; 0):

$$V^{0}(1;0) = \min \quad V(1;0)F(1; \; {}_{F(1;0)} \qquad 0) + V(0;0)F(0;0) \qquad ;1 \,: \qquad (8)$$

The minimum takes care of the case where this sum is larger than 1. Should we nd that $V^{0}(1;0)$ is less than one, let the remainder of V^{0} be de ned as $V^{0}(0;1) = V(0;1)$ and $V^{0}(0;0) = 0$. Otherwise, $V^{0}(1;0) = 1$, and we repeat the above exercise by moving mass from $V(0;0)$ to $V^{0}(0;1)$, that is $V^{0}(0;1)$ is the excess mass from $V(0;0)$ left over after maximizing $V^{0}(1;0)$:

$$V^{0}(0;1) = \min \qquad {}^{V}F(0;1) \qquad V(0;0)F(0;0) \;\; (1 \qquad (1;0))F(1;0) + V(0;1)F(0;1) \qquad ;1 \,: \qquad (9)$$

Again, if $V^{0}(0;1)$ is less than 1, then let $V^{0}(0;0) = 0$. On the other hand, if $V^{0}(0;1) = 1$, then:

$$V^{0}(0;0) = \frac{V(0;0)F(0;0) + V(1;0)F(1;0) + V(0;1)F(0;1) \;\; F(0;1) \;\; F(1;0)}{F(0;0)} \,: \qquad (10)$$

We have now de ned V^{0}, our potential solution. We proceed by proving that V^{0} satis es both of the incentive constraints denoted IC_1 and IC_2. This will follow because V satis es both incentive constraints and V^{0} is weakly more convincing than V was. We previously mentioned that the V^{0} mechanism entails the same components of the V mechanism. The excess \weight" that $V(0;0)$ carried as a result of it being bigger than 0 was transferred to the other states that are satisfying to voters 1 and 2. Therefore, it must be the case that $P_1(V^{0})$ and $P_2(V^{0})$ are larger than $P_1(V)$ and $P_2(V)$. However, we are constrained by the fact that neither mechanism can be above 1. Therefore, in the best case scenario, V^{0} is equivalent to 1, which is the maximum the V mechanism can be as well. Thus, V^{0} must also satisfy the same incentive constraints that satis es. We will rst show that V^{0} satis es voter 1's incentive constraint. As before, we use

$n_1(V) = F(1;1) + V(1;0)F(1;0)$ to denote the numerator of $P_1(V)$ and $d_1(V) = V F$ to denote the denominator of $P_1(V)$. There are multiple di erent cases depending upon the realized values of V^{0}, we begin with the case where $V^{0}(1;0) = 1$ and $V^{0}(0;1) < 1$.

$$P_1(V^{0}) = \frac{(V^{0} \; F \, j \; 2ff1;1g;f1;0gg}{V^{0} \;\; F}$$

$$= \frac{V^{0}(1;1)F(1;1) + V^{0}(1;0)F(1;0)}{}$$

V^0 F

F(1;1) + F(1;0)

=

V^0 F

F (1; 1) + V (1; 0)F (1; 0) + (1 V (1; 0))F (1; 0)

=

V^0 F

F (1; 1) + V (1; 0)F (1; 0) + (1 V (1; 0))F (1; 0)

= V^0(1; 1)F (1; 1) + V^0(1; 0)F (1; 0) + V^0(0; 1)F (0; 1) + V^0(0; 0)F (0; 0)

F (1; 1) + V (1; 0)F (1; 0) + (1 V (1; 0))F (1; 0)

= F(1; 1) + F(1; 0) + V^0(0; 1)F(0; 1) + V^0(0; 0)F(0; 0)

F (1; 1) + V (1; 0)F (1; 0) + (1 V (1; 0))F (1; 0)

= F (1; 1) + V (1; 0)F (1; 0) + (1 V (1; 0))F (1; 0) + V^0(0; 1)F (0; 1) + V^0(0; 0)F (0; 0)

(1 V (1; 0))F (1; 0) + n_1(V)

=

d_1(V)

The line of reasoning follows that of Remark 1 made prior to Lemma 1. P^0(V) is greater than P (V). Therefore, V^0 must also satisfy the incentive constraints of both voters. Furthermore, since $^P V^0$(a; b)F (a; b) = ^{P}V (a; b)F (a; b) the candidate receives the same total number of votes. Therefore, if V is optimal, V^0 must be optimal as well. □

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As a converse to Lemma 2, if V (0; 0) > 0 and for = 0; V ( ) < 1, then it must be the case that V ( ) could be increased. Furthermore, it could be increased by more than the corresponding decrease in V (0; 0) because the cost of convincing a voter in (0; 0) is greater than that of convincing a voter in .

Critically, neither voter is satis ed in (0; 0), in a sense, it is more costly to convince a voter in (0; 0) than any other state. Therefore, any \weight" that V (0; 0) carries, i.e. when V (0; 0) > 0, should be distributed to the other political states to bene t the candidate without violating incentive constraints and thereby improve the mechanism. Lemma 2 takes into consideration hypothetical scenarios in distributing the \weights" so the other subcomponents of the V mech-anism don't exceed 1 in probability, and increase the frequency with which both voters vote. This is achieved by transferring the excess \weight" from V (0; 0) to the political states favored

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by the voters. For example, if $V(1;1) < 1$, the \weight" could be shifted to this political state, as it is favored by both voters and will thus make the voters vote more frequently, as proven by Lemma 1. Lemma 3 codi es this intuition upon this notion through showing when the excess weight from (0; 0) can be redistributed.

Lemma 3. Let $N = 2$, $V(0;0) > 0$ and either

- $V(1;0); V(0;1) < 1$
- $P_1(1) <\ _1$ where 1 is the mechanism that tells voters to always vote and $V(1;0) < 1$

then V is not optimal.

Proof. Note if = (1; 1), then the converse of Lemma 1 implies that V is not optimal imme-diately. Then, without loss of generality, let = (1; 0). Since V was optimal, it must be the

case that increasing $V(1;0)$ directly violates an incentive constraint. Otherwise, V could be replaced by a V^0 where $V^0(1;0) > V(1;0)$ and $V^0( ) = V( )$ for 6= (1; 0) where V^0 increases

the probability of the candidate's election. However, since $P_1(V)$ is increasing in $V(1;0)$, it cannot be the case that voter 1's incentive constraint is violated by increasing $V(1;0)$. Thus, increasing $V(1;0)$ must violate voter 2's incentive constraint.

If $V(1;0); V(0;1) < 1$, then we can reduce $V(0;0)$ and increase $V(1;0)$ and $V(0;1)$ simulta-neously to strictly increase $P_1(V)$ and $P_2(V)$. Then, the voter's incentive constraints must be loosened, therefore the decrease in $V(0;0)$ is less than the corresponding total increase in $V(1;0)$ and $V(0;1)$. Then, neither incentive constraint is violated, but the candidate is elected strictly more often. Therefore, V was not optimal.

If $P_1(1) <\ _1$ where 1 is the mechanism that tells voters to always vote and $V(1;0) < 1$, we can then construct V^{00} in a manner similar to the construction in Lemma 2 through decreasing

$V(0;0)$ and increasing $V(1;0)$ by the same amount.

$$V^{00}(1;0) = \min \frac{V(1;0)F(1;0) + V(0;0)F(0;0)}{F(1;0)} ; 1 : \qquad (11)$$

$$V^{00}(0;0) = \frac{V(0;0)F(0;0) + V(1;0)F(1;0) \qquad V^{00}(1;0)F(1;0)}{F(0;0)} : \qquad (12)$$

By the same argument as in the proof of Lemma 2, this does not violate either voter's incentive constraint. The general premise of the argument in Lemma 2 revolved around the usage of excess \weight" accumulated in V (0; 0), namely V (0; 0) > 0. We expand upon this argument to prove Lemma 3. The excess voting \weight" from V (0; 0) is moved to state (1; 0). By default, the candidate is elected to the same degree, but voter 1's incentive constraint is strictly weakened.

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Furthermore, voter 1's incentive constraint is strictly loosened, since $P_1(V") > P_1(V)_1$. To see this formally, consider the following:

$$P_1(V^{00}) = \frac{(V^{00} F j \; 2ff1;1g;f1;0gg}{V0_F}$$

$$= \frac{V^{00}(1;1)F(1;1) + V^{00}(1;0)F(1;0)}{V^{00} F}$$

$$= \frac{F(1;1) + V^{00}(1;0)F(1;0)}{V^{00} F}$$

$$= \frac{F(1;1) + V^{00}(1;0)F(1;0)}{V^{00}(1;1)F(1;1) + V^{00}(1;0)F(1;0) + V^{00}(0;1)F(0;1) + V^{00}(0;0)F(0;0)}$$

$$= \frac{F(1;1) + V(1;0)F(1;0) + V(0;0)F(0;0)}{F(1;1) + V(1;0)F(1;0) + V(0;0)F(0;0) + V(0;1)F(0;1)}$$

$$= \frac{V(0;0)F(0;0) + n_1(V)}{d_1(V)}$$

$$> P_1(V)$$

Thus, $P_1(V^{00}) > P_1(V)$, and therefore $P_1(V) ; P_1(V^{00}) >$. Then, V (1; 0) can be increased by a relatively greater amount than V (0; 0) was decreased without violating voter 1's incentive constraint, due to the slackness introduced into the constraint. Therefore, the total probability of election is increased for the candidate. Thus, V could not have been optimal. □

Lemma 3 illustrates a speci c situation in which the \weight" of state V (0; 0) is shifted to more convincing states. Similarly to Lemma 2, neither voter is satis ed at state V (0; 0), making it advantageous from the point of view of the candidate to increase the frequency to which he or she tells the voters to vote for their favored political state. Importantly, under the assumptions present in Lemma 3, the candidate then can use the excess convincing power generated to increase the probability with which the voters are persuaded. We have shown that

V is not optimal under the given conditions, as P_1(V ") is greater than P_1(V). As per the remark prior to Lemma 1, this implies that V " F > V F , therefore making V not optimal, proving 3.

Theorem 1. Let N = 2, then an optimal mechanism V satis es the following:

V(1;1) = 1

1. If $_i$ = 1 for both voters:

   The optimal mechanism sets the remaining values of V as follows, V ( ) = 0; 8 6= (1; 1).

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2. If $_1$ = 1; $_2$ < 1:

   V(0;1) = V(0;0) = 0.

$$V(1;0) = \min {}^{F} \quad \frac{{}_2F(1;0)^{2}}{(1;1)(1\qquad)} \quad ;1 \qquad (13)$$

3. If $_1; _2 < 1$ and under V (1; 1) = V (1; 0) = V (0; 1) = 1; V (0; 0) = 0 we have $P_i(V)$ $_i$ for both i then:

$$V(0;0) = \frac{(F(1;1) \quad + F(1;0))(1 \quad _1) \quad _1F(0;1)}{_1F(0;0)} \qquad (14)$$

4. Otherwise:

$$V(1;0) = \frac{(1 \qquad _2)F(1;1)}{(1+ \ _1 + \ _2)F(1;0)}$$

$$V(0;1) = \frac{(1 \qquad _1)F(1;1)}{(1+ \ _1 + \ _2)F(0;1)}$$

V(0;0) = 0

Proof. First, if both voters require certainty, i.e. $_1 = _2 = 1$, then the process of nding the optimal mechanism is trivial. V (1; 1) = 1; V ( ) = 0 for 6= (1; 1) as any mechanism that involves V ( ) > 0 fails to ever result in the candidate's election. Furthermore, if V (1; 1) < 1, then Lemma 1 is violated.

Next, suppose only one voter required certainty. Without loss of generality, let $_1 = 1 > _2$. Then, it immediately follows that V (1; 1) = 1; V (0; 0) = V (0; 1) = 0. To determine V (1; 0), we will need to ensure that voter 2 is convinced. That is, voter 2's incentive constraint is the binding constraint for V (1; 0). $P_2(V) = \frac{F(1;1)}{F(1;1)+V(1;0)F(1;0)} \quad _2$ is the constraint in that case. Then, solving for V (1; 0) implies that V (1; 0) = $\frac{F(1;1)(1 \quad _2)}{_2F(1;0)}$. Of course, if voter 2 has su ciently low standards, i.e. $_2$ is low, this term may be greater than 1, so we set V (1; 0) = minf $\frac{F(1;1)(1 \quad _2)}{_2F(1;0)}$; 1g.

Now, let both $_1; _2 < 1$, and we focus on the general case. Similar to the argument in the proof of Lemma 3, we note that the candidate will only resort to convincing voters in state (0; 0) if she has fully convinced them in every other state. Then, there are two subcases, one where voters vote even when V (1; 1) = V (1; 0) = V (0; 1) = 1 and the other where the voters don't.

First, suppose the mechanism with V (1; 1) = V (1; 0) = V (0; 1) = 1 satis es both voter's incentive constraints. Then, V (0; 0) will be increased to the maximal extent, namely until it

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violates a voter's incentive constraint. Voter 1's IC constraint is violated when $_1 = P_1 =$

F(1;1)+F(1;0)

. Inverting this and solving for V (0; 0) implies that:

F(1;1)+F(1;0)+F(0;1)+V (0;0)F(0;0)

$$V(0;0) = \frac{(F(1;1) + F(1;0))(1 \quad _1) \quad _1F(0;1)}{_1F(0;0)}$$

Similar algebra solving for when voter 2's IC constraint binds implies that:

$$V(0;0) = \frac{(F(1;1) + F(0;1)(1 \quad _2) \quad _2F(1;0)}{_2F(0;0)}$$

Taking the minimum of these two functions and 1 generates the optimal value of V (0; 0). Last, suppose V (1; 1) = V (1; 0) = V (0; 1) = 1 doesn't convince both voters. Then, Lemma 3

implies that V (0; 0) = 0. We proceed by using a simple Lagrangian to nd the optimal solution.

$$L = V \;\; F \quad [P_1(V) \quad _1] \quad [P_2(V) \quad _2] \qquad (15)$$

Taking the derivative of Equation 15 with respect to the shadow variables recovers the incentive constraints. At a minimum, one of the two must bind by assumption. If neither was binding, then a mechanism with V (1; 1) = V (1; 0) = V (0; 1) = 1 would have convinced both voters. Without loss of generality, let voter 1's incentive constraint bind. Then, solving $P_1(V) = {}_1$ for V (0; 1) implies that:

$$V(0;1) = \frac{(1 \quad _1)[F(1;1) + V(1;0)F(1;0)]}{_1F(0;1)} \qquad (16)$$

If voter 2's incentive constraint fails to bind, = 0, then V (1; 0) = 1 to maximize voter 1's willingness to vote. This implies V (0; 1) = $\frac{(1 \quad _1)[F(1;1)+F(1;0)]}{}$. Note, that if voter 2's incentive

$$_1F(0;1)$$

constraint binds and voter 1's IC constraint does not, the outcome is symmetric.

Suppose instead that both incentive constraints bind. We can then solve for V (1; 0) using voter 2's incentive constraint, $P_2(V) = {}_2$. Some algebra then implies:

$$V(1;0) = \frac{(1 \quad {}_2)F(1;1)}{(1+ \ {}_1 + \ {}_2)F(1;0)}$$

$$V(0;1) = \frac{(1 \quad {}_1)F(1;1)}{(1+ \ {}_1 + \ {}_2)F(0;1)}$$

Then, we have considered an exhaustive list of possible cases and found the optimal solution in each of them. Therefore, the mechanisms de ned in the theorem above are optimal under each condition presented.

□

5 Conclusion

We have proven several novel insights through our analysis. First, we focus on the di erences regarding private and public information. We analyzed information asymmetry in this paper under the conditions that the information the candidate was providing was public. She could not send individualized messages to sway individual voters. Had she been able to do so, her optimal mechanism would have changed considerably. In particular, she would always have been able to secure the vote up to the most reluctant voter's threshold. While the degree to which this impacts the results of this research is inconspicuous at a super cial level, the details of the ndings indicate that the results could be drastically di erent in a paradigm that allows for candidates to perpetuate privatized information. Privatized information reduces the burden on the candidate to be consistent in his or her messaging. There is no mechanism of accountability for potentially contradictory statements that could be made in order to persuade voters to vote within the candidate's interest that would exist in a public setting.

However, this implies that the voter's themselves act as a restriction on the set of messages a candidate can provide. Even in the absence of formal accountability measures, if voters share the statements they have received from the candidate, an informal check is placed on the candidate. On the other hand, if certain subgroups of voters believe that they have received the \true" message from a candidate, and other voters are being deceived, such a behavioral bias could allow the candidate to take advantage of voters. Given that we categorized the candidate's actions depending on various political states, this makes the joint distribution over individual voter's state regarding the candidate's choice to convey information obsolete.

The correlation of voters' beliefs is also critical in this setting. In this paper, we examined the voters as independent beings, incapable of contaminating or swaying each other's decisions in a binary voting situation. Therefore, the sole variable in conveying information is the candidate's message itself. Accounting for the degree to which voters in uence each other's behavior would add another layer of complexity to the problem of information asymmetry that is not fully explored in this paper. However, in large scale elections, it is unlikely voters focus on being pivotal. For example, third parties routinely receive up to 5% of the vote, which would clearly be irrational if a voter assuming themselves to be pivotal, since any individual voter is in nitesimally likely to tip the scales in favor of a third party.

Through modelling the relationship between a political candidate and its voting constituents, we can apply these ndings to broader contexts. For instance, a board member in a company requiring support for her ideas or business strategies may bene t from understanding the con-straints posed to her in order to achieve success and ful ll her goal. Attempting to convince everyone may be a fool's errand if their interests are at odds. Instead, we provide insight regarding which of her fellow members she should attempt to convince in order to maximize her odds of success.

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